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SB-BACO4
A Gompertzian Discrete Model for Tree Competition

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Abstract. For interspecific competition between tree populations, the author proposes a discrete Gompertzian model named SB-BACO4. The model is a direct extension of a discrete form of the Gompertz equation. Both deterministic and stochastic versions are exhibited. The model is capable of the correct recognition of six distinct patterns of tree interaction, previously defined by the author. He uses model SB-BACO4 to establish the simulator SB-PINHARDI for mixed stands of *Quercus robur+Pinus pinaster* and *Fraxinus excelsior+Pinus pinaster*. In appendices, the author presents the listing of the program, written in BASIC, and a sample of its output. A general Gompertzian explicit model for tree competition is also presented (SB-BACO5).
Key words: discrete model; general model for plant competition; interspecific tree competition; simulation of mixed stands; stochastic model; *Fraxinus excelsior*; *Pinus pinaster*; *Quercus robur*

Sumário. O autor apresenta o modelo discreto SB-BACO4, para a competição inter-específica entre populações de árvores. O modelo é uma extensão de uma forma discreta da equação de Gompertz, e são propostas as suas versões determinísticas e estocásticas. O modelo é capaz de descriminar coerentemente seis padrões de interacção entre populações de árvores, anteriormente definidos pelo autor. O modelo SB-BACO4 é aplicado no simulador SB-PINHARDI para povoamentos de *Quercus robur+Pinus pinaster* e *Fraxinus excelsior+Pinus pinaste*r. Em apêndices, exibem-se a listagem do simulador, escrito em BASIC, e uma amostra da sua saída. O autor também apresenta um modelo geral, Gompertziano e explícito, para a competição entre árvores (SB-BACO5).
Palavras-chave: competição inter-específica entre árvores; modelo discreto; modelo geral para a competição; modelo estocástico; simulação de povoamentos mistos; *Fraxinus excelsior; Pinus pinaster; Quercus robur*

Résumé. L'auteur présente le modèle discret SB-BACO4 pour la compétition inter-spécifique parmi des populations d'arbres. Le modèle est une extension de la forme discrète de l'équation de Gompertz. Le modèle SB-BACO4 a une version déterministe et une autre stochastique. Le modèle est capable de discriminer correctement six patrons d'interaction parmi deux populations d'arbres. Le modèle est appliqué dans le simulateur SB-PINHARDI pour les peuplements mixtes avec *Quercus robur+Pinus pinaster* et *Fraxinus excelsior+Pinus pinaster.* Le logiciel du simulateur, écrit en BASIC, est présenté. L'auteur propose aussi un modèle général Gompertzien pour la compétition parmi des populations d'arbres (SB-BACO5).
Mots clés: compétition inter-spécifique parmi les arbres; modèle discret; modèle général pour

la compétition; modèle stochastique; simulation des peuplements mixtes; *Fraxinus excelsior; Pinus pinaster; Quercus robur*

Introduction

In my systematic approach to the structure and dynamics of tree populations, I first approached pure, and after mixed stands, both even--aged (BARRETO, 1997, 1998a, 1999a,b,c, 2000b, 2001) and uneven--aged (1998b). Within these elaborations, for self-thinned even-aged mixed stands (SEMS), I already established three models.

In BARRETO (1997, 1999a; 2001) I presented model BACO3, for SEMS. This model is a direct extension of the differential form of the Gompertz equation, as the Lotka-Volterra model for competition is a direct extension of the logistic equation. In this line of research, in this paper, I will attempt to establish a Gompertzian discrete model for tree competition.

In the next sections, I will establish model SB-BACO4, and I will evaluate its performance. In simulator SB-PINHARDI, written in BASIC, I will apply model SB-BACO4 to SEMS of *Pinus pinaster*+*Fraxinus excelsior* (PP+FE), and of *Pinus pinaster*+*Quercus robur* (PP+QR). In the Appendices, I present the listing of the simulator, written in BASIC, and a sample of its output. I will introduce a general Gompertzian explicit model for tree competition, and after I close the article with adequate conclusive remarks.

Model SB-BACO4

The Gompertz equation can be written as:

$$y(t) = y_f \, Ry^{\exp(-c(t-t_0))} \qquad (1)$$

where $\mathbf{y_f}$ is the final value of **y**, **Ry** is the ratio $\mathbf{y_0/y_f}$, being $\mathbf{y_0}$ the value of **y** at age $\mathbf{t_0}$, when the Gompertzian process starts. **c** is a constant. **Ry** and **c** are constant for a given tree species.

A differential form of the Gompertz equation can have the following form:

$$dy/dt = c\, y \ln y_f (1 - \ln y/\ln y_f) \qquad (2)$$

A discrete form for the Gompertz equation can be written as (e.g., DENNIS and TAPER, 1994):

$$y(t+1) = y(t) \exp(a - c \ln y(t)) \qquad (3)$$

In eq. (3), the carrying capacity corresponding to the asymptote is $y_f=\exp(a/c)$. The value of the constant **a** depends on the size of the population. The larger is the number of trees, the larger is the value of **a.**

An expression for **a** is the following one:

$$a= b + c \ln y_0 \qquad (4)$$

For easier comparisons, I use the same notation I employed with model BACO3 (BARRETO, 1999a). Assuming two tree populations, **M** and **N**, described by their number of trees per hectare (or standing volume, or tree biomass), I propose model SB-BACO4 (MB4) for interspecific tree competition, being this model a direct extension of eq. (3).

I write MB4 as follows:

$$M(t+1)=M(t) \exp(a_1 - c_1 \ln M(t) + n \ln N(t)) \qquad (5a)$$

$$N(t+1)=N(t) \exp(a_2 - c_2 \ln N(t) + m \ln M(t)) \qquad (5b)$$

being **m** and **n** the coefficients of competition (CC).

Environmental stochasticity can be introduced in MB4 as follows:

$$M(t+1)=M(t)\exp(a_1 - c_1 \ln M(t) + n \ln N(t) + w_1 Z(t)) \quad (6a)$$

$$N(t+1)=N(t)\exp(a_2 - c_2 \ln N(t) + m \ln M(t) + w_2 Z(t)) \quad (6b)$$

where $\mathbf{w_1}$ and $\mathbf{w_2}$ are positive constants and $\mathbf{Z(t)}$ has normal distribution with mean of 0 and variance of 1.

Obviously, demographic stochasticity is precluded from the process of self-thinning. On the other hand, models with demographic stochasticity behave as deterministic when populations are large (DENNIS, MUNHOLLAND, and SCOTT, 1991).

The two forms of MB4 (eqs. (5), (6)) can be easily extend to more then two populations.

MB4 can be transformed to a logarithmic scale as follows:

$$LM(t+1)= LM(t) + a_1 - c_1 LM(t) + n LN(t) + w_1 Z(t) \quad (7a)$$

$$LN(t+1)= LN(t) + a_2 - c_2 LN(t) + m LM(t) + w_2 Z(t) \quad (7b)$$

where LM(t)=ln M(t), LN(t)=ln N(t), LM(t+1)=ln M(t+1), and LN(t+1)=ln N(t+1).

The Performance of Model SB-BACO4

To evaluate the performance of MB4, I used eqs. (5) to calculate the CC for the data I used in BARRETO (1999a), to typify the patterns of tree interaction. In BARRETO (1999a), for this purpose, I used six mixed stands, identified as Types I to VI, as exhibited in table 1. In table 2, I display the ratio of the relative growth rates of the couple of species of each mixed stand. In Appendix A, I exhibit the CC generated by MB4.

Specifically referring to the CC generated by MB4, their absolute values are smaller then the ones associated to model BACO3 (BARRETO, 1999a), but the general configuration of the patterns are reproduced. With this model, in SEMS PM+PS, for a short period of time, mutual antagonism (-,-) occurs (table 6.A, ages 20, and 30). Model BACO3 did not depict this interaction, as registered in BARRETO (1999a). Probably this is a consequence of the discretization of the continuous process of plant competition. When I used a less precise method of calculation with model BACO3, in BARRETO (1997), mutual antagonism also occurred.

Table 1 - Acronyms, species, and the simulated SEMS used to evaluate MB4

A. Species selectes	
AR= *Alnus rubra*	PP= *Pinus pinaster*
FE= *Fraxinus excelsior*	PM= *Pseudotsuga menziesii*
PC= *Pinus contorta*	
PE= *Pinus elliotti*	PS= *Picea sitchensis*
PH= *Pinus halepensis*	QR= *Quercus robur*
B. Mixtures (SEMS)	
QR+PP, PC+FE, PE+PH, PH+PP, AR+PM, PM+PS	
The first species is the one that behaves as dominant	
C. Coefficients of competition in tables 1A to 6A	
m.x measures the effect of the second species upon the first species, when the fraction of the number of trees of the latter, at age 10, is 0.x. **n.x** evaluates the effect of the first species upon the second species, when the fraction of the number of trees of the former, at age 10, is 0.x	

Table 2 - The ratios of the relative growth rates of the couples of species used to establish a tentative typification of the patterns of tree interaction with model BACO3, and reused now with model SB-BACO4. See Appendix A

Age	QR/PP Type I	PC/FE Type II	PE/PH Type III	PH/PP Type IV	AR/PM Type V	PM/PS Type VI
10	2.209	1.148	2.368	1.545	1.156	0.987
20	2.417	Constant	2.345	1.122	1.122	1.007
30	2.645		2.321	0.815	1.089	1.027
40	2.894		2.298	0.592	1.056	1.048
50	3.167		2.276	0.430	1.025	1.069
60	3.465		2.253	0.312	0.995	1.091
70	3.791		2.231	0.226	0.965	1.112
80	4.148		2208	0.164	0.937	1.135
90	4.539		2.186	0.119	0.909	1.158

Simulator SB-PINHARDI

As already said, simulator SB-PINHARDI (SPD) covers the SEMS PP+QR, PP+FE. These two stands show interactions of type I. Thus, the ratio "relative growth rate of the hardwood/relative growth rate of PP" is greater than one and increases with age. The hardwood is clearly the dominant species, in the mixture.

SPD applies eqs. (5), (6), as it has an option to simulate a quasi-stochastic process of self-thinning. Eqs. (6) can provoke not only the decrease of the number of trees but also its increasing. As self-thinned trees cannot resuscitate, SPD has a procedure to avoid the stochastic increase of the number of trees. When the increase occurs, it applies the deterministic calculation of self-thinning. Thus, as the process is not totally stochastic, I call it quasi-stochastic.

To establish the values of $\mathbf{a_i}$ and $\mathbf{c_i}$, in eqs. (5), I fitted eq. (4) to data simulated with eq. (1), and values of $\mathbf{y_0}$ of 1000, 2000, 10000 trees/ha. The values of the constants are exhibited in table 3.

Table 3 - Values of the constants in eq. (4). All coefficients of determination are equal to one

Species	b	c
BB+PP		
QR	-0.19426	0.04017
PP	-0.08754	0.04877
FE+PP		
FE	-0.13298	0.03729
PP	-0.08753	0.04877

To obtain the simulated data to fit eq. (4), in eq.(1), the values of **c** I used were 0.041, 0.038, 0.050, respectively, for QR, FE, PP.

To obtain the values of the CC, I used the following equation:

$$m(t) \text{ or } n(t) = d + f \ln x_1(t) + g \ln x_2 \qquad (8)$$

where $\mathbf{x_1(t)}$ is the total density of the SEMS at age **t**, and $\mathbf{x_2}$ is the fraction of trees of the hardwood, at age 10. To fit eq. (8), I simulated two complete additive designs of the two SEMS, with model BACO2. In the SEMS FE+PP, I used the frequencies 1000, 3400, 6800, 8200. In the SEMS QR+PP, I used the frequencies 1000, 4000, 7000, 10000.

The values of the constants in eq. (8) are displayed in table 4.

Table 4 - Values of the constants in eq. (8)

CC	d	f	g	R^2
QR+PP				
m	0.024252	-0.002357	-0.008671	0.901
n	-0.019991	0.001429	-0.004050	0.959
FE+PP				
m	0.012768	-0.001423	-0.005044	0.864
n	-0.014185	0.001079	-0.002929	0.949

SPD has two subprograms FRAPIN (for SEMS FE+PP) and ROPIN (QR+PP). For each species, at age 10, the simulator asks for the number of trees per hectare and the volume of the mean tree (c.m.). For ages 10, 20, ... 80, SPD prints the number of trees per hectare, and the standing volume (c.m./ha) of each tree population, in the mixture.

Besides the choice of the mixture (QR+PP or FE+PP) the user can also choose a deterministic or a quasi-stochastic simulation.

Comparing the values generated SPD with those simulated by model BACO2, SPD is less accurate (errors greater than 10%) for very high total densities or smaller fractions of one of the competitors (weak competition). The critical values of these two parameters are indicated in the listing of SPD, exhibited in the Appendix B. A sample of the output of the simulator is presented in Appendix C.

In eqs. (6), SPD assumes w_1=0.4 (hardwood population), and w_2=0.5 (pine population).

The "rem" comments I introduced in the listing of SPD make its structure conspicuous and clear. This listing can be easily converted into a Visual Basic program.

The Classical Approach

In this section let me approach MB4 when the CC are constant and negative. This is the usual analysis presented, in textbooks of ecology, for the Lotka--Volterra and other models, for competition.

With these assumptions, the outcomes of competition can be described as follows:

- Case I: $1/m<a_1/a_2>n$ Species M wins; species N is wiped out.
- Case II: $1/m>a_1/a_2<n$ Species N wins; species M is wiped out.
- Case III: $1/m>a_1/a_2>n$ Both species coexist in stable equilibrium.
- Case IV: $1/m<a_1/a_2<n$ Both species persist in unstable equilibrium.

The sizes of the populations in equilibrium are:

$$M^*=\exp((a_1-na_2/c_2)/(c_1-mn/c_2)) \quad (9)$$

$$N^*=\exp((a_2-ma_1/c_1)/(c_2-mn/c_1)) \quad (10)$$

A General Model for Tree Competition

Given the previous elaborations, for a mixture of several species, I can introduce a general model for tree competition, as:

$$y_i(t+1) = A\,B \qquad i=1,2\ldots n \quad (11)$$

being $\mathbf{y_i}$ the number of trees (their standing volume or standing biomass) of species **i** in a SEMS, **A** is the Gompertzian variation of the same variable in pure stand, at age **t,** and **B** is the effect of the other competitors. The effect of the competitors is estimated as:

$$B= \exp [\Sigma (\alpha_{i,j} \ln y_j(t))] \qquad i \neq j \quad (12)$$

where $\boldsymbol{\alpha_{i,j}}$ measures the effect (constant or variable) of species **j** upon species **i.** For instance, **A** can be obtained using eq. (1), as an alternative to eq. (3), and an algebraic Gompertzian continuous model for tree competition is made available.

For easy reference, I call this model SB-BACO5.

Let me introduce the classical analysis (n=2, both CC are constant, and negative) when eq. (1) is used in eq. (11):

- Case I: $1/|\alpha_{2,1}| < \ln y_{1f}/\ln y_{2f} > |\alpha_{1,2}|$. Species y_1 wins; species y_2 is wiped out.
- Case II: $1/|\alpha_{2,1}| > \ln y_{1f}/\ln y_{2f} < |\alpha_{1,2}|$. Species y_2 wins; species y_1 is wiped out.
- Case III: $1/|\alpha_{2,1}| > \ln y_{1f}/\ln y_{2f} > |\alpha_{1,2}|$. Both species coexist in stable equilibrium.
- Case IV: $1/|\alpha_{2,1}| < \ln y_{1f}/\ln y_{2f} < |\alpha_{1,2}|$. Both species persist in unstable equilibrium.

The sizes of the populations at equilibrium are:

$$y_1^* = \exp((\ln y_{1f} + \alpha_{1,2} \ln y_{2f}) / (1 - (\alpha_{1,2}\,\alpha_{2,1}))) \quad (13)$$

$$y_2^* = \exp((\ln y_{2f} + \alpha_{2,1} \ln y_{1f}) / (1 - (\alpha_{1,2}\,\alpha_{2,1}))) \quad (14)$$

These results, coherently, are also applicable to model BACO3. Formally, they are similar to the classical analysis for model BACO4, previously displayed.

Final comments

Model SB-BACO5, eqs. (11)-(12), is a general Gompertzian algebraic model for plant competition that clearly relates the growth of the species in pure stand to the effect of the competitors. It is also enough flexible to accommodate any integral or difference form of the Gompertz equation. I admit that model SB-BACO5 has both theoretical and pratical interest, as MB4 illustrates. The family of models, I introduced here, broaden the comprehensiveness of my theory for mixed stands and it illustrates its versatility, internal coherence, and fruitfulness. Also, it allows, in a straight manner, the simultaneous estimation of the trajectories of the competitive populations both in pure and mixed stands, and their easy comparisons.

My evaluation of the predictability of MB4 showed that the model is more adequate to reproduce data from situations where competition is clear (types I, II, III), as verified with SPD. Its performance is less accurate when applied to stands where the species have coevolved (shifts of dominance) and/or have very close competitive ability (particularly, types IV, V, VI).

I admit that MB4 may open new opportunities to the analysis of the time series of mixed stands.

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Appendix A. CC of the six types of tree interaction, obtained with MB4

Table 1A – The CC for the SEMS of QR+PP (Type I)

Age	m.2	n.2	m.5	n.5	m.8	n.8
10	0.011650	-0.001543	0.008384	-0.003807	0.003371	-0.005270
20	0.013109	-0.002127	0.009979	-0.004787	0.004571	-0.006464
30	0.014249	-0.002517	0.011333	-0.005415	0.005652	-0.007312
40	0.015076	-0.002730	0.012330	-0.005849	0.006439	-0.007942
50	0.015656	-0.002856	0.013031	-0.006137	0.006984	-0.008387
60	0.016053	-0.002929	0.013508	-0.006321	0.007348	-0.008690
70	0.016320	-0.002969	0.013826	-0.006436	0.007588	-0.008889
80	0.016497	-0.002991	0.014035	-0.006507	0.007743	-0.009019
90	0.016613	-0.003062	0.014170	-0.006549	0.007843	-0.009102

Table 2A - The CC for the SEMS of PC+FE (type II)

Age	m.2	n.2	m.5	n.5	m.8	n.8
10	0.001582	-0.000413	0.001573	-0.001012	0.000443	-0.001400
20	0.001680	-0.000584	0.001628	-0.001280	0.000394	-0.001691
30	0.001868	-0.000607	0.001808	-0.001383	0.000489	-0.001823
40	0.002018	-0.000626	0.001957	-0.001469	0.000567	-0.001931
50	0.002132	-0.000641	0.002073	-0.001537	0.000629	-0.002015
60	0.002216	-0.000653	0.002161	-0.001589	0.000676	-0.002078
70	0.002277	-0.000661	0.002226	-0.001628	0.000711	-0.002124
80	0.002321	-0.000667	0.002273	-0.001655	0.000736	-0.002157
90	0.002352	-0.000671	0.002307	-0.001674	0.000754	-0.002180

Table 3A - The CC for the SEMS of PE+PH (type III)

Age	m.2	n.2	m.5	n.5	m.8	n.8
10	0.020394	-0.002571	0.014678	-0.006274	0.005934	-0.008663
20	0.021372	-0.003451	0.016612	-0.007499	0.007915	-0.010022
30	0.022326	-0.003664	0.017930	-0.008025	0.008929	-0.010927
40	0.022855	-0.003737	0.018600	-0.008278	0.009386	-0.011341
50	0.023110	-0.003767	0.018913	-0.008394	0.009590	-0.011536
60	0.023226	-0.003779	0.019054	-0.008446	0.009680	-0.011624
70	0.023278	-0.003784	0.019117	-0.008469	0.009719	-0.011663
80	0.023301	-0.003786	0.019145	-0.008479	0.009737	-0.011681
90	0.023311	-0.003787	0.019157	-0.008484	0.009745	-0.011688

Table 4A - The CC for the SEMS of PH+PP (type IV)

Age	m.2	n.2	m.5	n.5	m.8	n.8
10	0.004207	-0.000807	0.003021	-0.002003	0.001201	-0.002779
20	0.002203	-0.000433	0.001589	-0.000978	0.000599	-0.001314
30	0.001289	-0.000096	0.000936	-0.000178	0.000353	-0.000222
40	0.000869	0.000126	0.000631	0.000345	0.000235	0.000481
50	0.000682	0.000269	0.000493	0.000671	0.000180	0.000914
60	0.000601	0.000359	0.000431	0.000870	0.000154	0.001174
70	0.000565	0.000414	0.000403	0.000990	0.000142	0.001177
80	0.000550	0.000447	0.000391	0.001062	0.000136	0.001178
90	0.000543	0.000467	0.000385	0.001105	0.000134	0.001179

Table 5A - The CC for the SEMS of AR+PM (type V)

Age	m.2	n.2	m.5	n.5	m.8	n.8
10	0.002453	-0.000645	0.001751	-0.001573	0.000680	-0.002174
20	0.002300	-0.000824	0.001639	-0.001790	0.000535	-0.002336
30	0.002339	-0.000791	0.001742	-0.001781	0.000613	-0.002298
40	0.002374	-0.000764	0.001828	-0.001773	0.000675	-0.002265
50	0.002402	-0.000744	0.001892	-0.001766	0.000720	-0.002239
60	0.002424	-0.000730	0.001939	-0.001760	0.000752	-0.002219
70	0.002439	-0.000720	0.001971	-0.001756	0.000773	-0.002204
80	0.002451	-0.000714	0.001993	-0.001752	0.000787	-0.002193
90	0.002458	-0.000710	0.002007	-0.001749	0.000796	-0.002186

Table 6A - The CC for the SEMS of PM+PS (type VI)

Age	m.2	n.2	m.5	n.5	m.8	n.8
10	-0.000208	0.000026	-0.000159	0.000107	-0.000179	0.000163
20	-0.000097	-0.000202	-0.000141	-0.000228	-0.000167	-0.000214
30	0.000149	-0.000218	0.000060	-0.000337	-0.000061	-0.000376
40	0.000338	-0.000232	0.000220	-0.000424	0.000024	-0.000499
50	0.000473	-0.000243	0.000339	-0.000488	0.000087	-0.000588
60	0.000565	-0.000250	0.000423	-0.000533	0.000132	-0.000648
70	0.000627	-0.000256	0.000480	-0.000564	0.000162	-0.000688
80	0.000667	-0.000259	0.000518	-0.000583	0.000182	-0.000714
90	0.000693	-0.000261	0.000542	-0.000596	0.000195	-0.000730

Appendix B. Listing of simulator SB-PINHARDI

```
CLS
? "                 SIMULATOR SB-PINHARDI"
?
? "A Gompertzian discret simulator for two self-thinned even-aged mixed"
? "           stands with Pinus pinaster and one hardwood"
? "                   © Luís Soares Barreto, 1999"
?:? "The stands considered are the following ones:"
?:? "    1=Pinus pinaster+Fraxinus excelsior"
?:? "    2=Pinus pinaster+Quercus robur"
?:?:INPUT "                Enter your choice (1, or 2)";ch
ON ch GOTO frap, rop
' ----------------------------------------------------------
' ----------------------------------------------------------
frap:
' 1.INITIALISE
' ----------------------------------------------------------
' 1.1 Program introduction and input of data
DIM a$(1)
CLS
?:? "                      SIMULATOR SB-PINHARDI"
? "                          PROGRAM FRAPIN"
? "A discrete simulator for self-thinned even-aged mixed stands of"
? "                 Fraxinus excelsior and Pinus pinaster"
?:? "                    (c) Luís Soares Barreto, 1999"
?:? "At age 10, I suggest to use:"
? "- Total densities less than 14000 trees/ha"
? "- Fractions of Fraxinus excelsior inside the range 0.3-.75"
?:? "At age 10, for Fraxinus excelsior introduce"
INPUT "   number of trees/ha";f
INPUT "   mean tree volume, c.m.";vf
?:? "At age 10, for Pinus pinaster introduce"
INPUT "   number of trees/ha";p
```

```
INPUT "   mean tree volume c.m.";vp
?:INPUT "         Do you want the stochastic simulation (y,n)"; a$
'-----------------------------------------------------------
' 1.2 Calculating constants
af = -.13298 + .03729 * LOG(f): ap = -.08753 + .04877 * LOG(p)
' ---------------------------------------------------------
' 1.3 Final tree volume
vff=vf/.00475:vpf=vp/.0677
' ---------------------------------------------------------
' 1.4 Standing volume at age 10
tvf=vf*f:tvp=vp*p
' ---------------------------------------------------------
' 1.5 Printing actual forest
?
LPRINT "                    SIMULATOR SB-PINHARDI"
LPRINT "                     PROGRAM FRAPIN"
LPRINT "Age: ";10
LPRINT "Fraxinus excelsior: ";f;" trees/ha     ";tvf;" c.m./ha"
LPRINT "Pinus pinaster: ";p;"  trees/ha     ";tvp;" c.m./ha"
LPRINT
' ---------------------------------------------------------
' 1.6 Preparing projection
r% = 11
y = LOG(f / (f + p))
' ---------------------------------------------------------
' ---------------------------------------------------------
' 2. PROJECTION
FOR t = 0 TO 70
 IF a$ = "n" THEN GOTO 5
  ' -------------------------------------------------------
' 2.1 Normal distributed variables with mean 0 and variance 1
 ri = 0
 FOR i = 1 TO 12
 cas = RND(3333): ri = ri + cas
 NEXT i
 ze1 = ri - 6
 ri = 0
 FOR i = 1 TO 12
 cas = RND(9999): ri = ri + cas
 NEXT i
 ze2 = ri - 6
  ' -------------------------------------------------------
 ' 2.2 Number of trees
 5 x = LOG(f + p)
 m = .012768 - .001423 * x - .005044 * y
 n = -.014185 + .001079 * x - .002929 * y
 zf=LOG(f):zp=LOG(p)
 e1=af - .03729 * zf + m * zp
 e2=ap - .04877 * zp + n * zf
 f2 = f * EXP(e1+.05*ze1)
```

```
 p2 = p *EXP(e2+.04*ze2)
 IF f2 > f THEN f2 = f * EXP(e1)
 IF p2 > p THEN p2 = p * EXP(e2)
 ' ---------------------------------------------------------
' 2.3 Tree volumes
 vf=vff*.00475^EXP(-.038*(t-1)):vp=vpf*.0677^EXP(-.05*(t-1))
 ' ---------------------------------------------------------
' 2.4 Standing volumes
 tvf=vf*f2:tvp=vp*p2
 ' ---------------------------------------------------------
' 2.5 Printing decennial projected values
 IF INT (r%/10)<>r%/10 THEN GOTO 15
 LPRINT "Age: ";r%
 LPRINT "Fraxinus excelsior: ";fix(f2);" trees/ha     ";tvf;"   c.m./ha"
 LPRINT "Pinus pinaster: ";fix(p2);" trees/ha     ";tvp;"       c.m./ha"
 LPRINT
 15 f = f2: p = p2: r% = r% + 1
NEXT t
END
' ---------------------------------------------------------
' ---------------------------------------------------------
rop:
' 1.INITIALISE
' ---------------------------------------------------------
' 1.1 Program introduction and input of data
DIM a$(1)
CLS
? "                       SIMULATOR SB-PINHARDI"
? "                            PROGRAM ROPIN"
? "A discrete simulator for self-thinned even-aged mixed stands of"
? "                     Quercus robur and Pinus pinaster"
?:? "                      (c) Luís Soares Barreto, 1999"
?:? "At age 10, I suggest to use:"
? "- Total densities between 3000 trees/ha and 17000 trees/ha"
? "- Fractions of Quercus robur inside the range 0.3-0.75"
?:? "At age 10, for Quercus robur introduce"
INPUT "    number of trees/ha";f
INPUT "    mean tree volume, c.m.";vf
?:? "At age 10, for Pinus pinaster introduce"
INPUT "    number of trees/ha";p
INPUT "    mean tree volume c.m.";vp
?:INPUT "          Do you want the stochastic simulation (y,n)"; a$
' ---------------------------------------------------------
' 1.2 Calculating constants
af = -.19426 + .04017 * LOG(f): ap = -.08754 + .04877 * LOG(p)
' ---------------------------------------------------------
' 1.3 Final tree volume
vff=vf/.0007:vpf=vp/.0677
' ---------------------------------------------------------
' 1.4 Standing volume at age 10
```

```
tvf=vf*f:tvp=vp*p
' ---------------------------------------------------------
' 1.5 Printing actual forest
?
LPRINT "                    SIMULATOR SB-PINHARDI"
LPRINT "                       PROGRAM ROPIN"
LPRINT "Age: ";10
LPRINT "Quercus robur: ";f;" trees/ha     ";tvf;" c.m./ha"
LPRINT "Pinus pinaster: ";p;"  trees/ha     ";tvp;" c.m./ha"
LPRINT
' ---------------------------------------------------------
' 1.6 Preparing projection
r% = 11
y = LOG(f / (f + p))
' ---------------------------------------------------------
' ---------------------------------------------------------
' 2. PROJECTION
FOR t = 0 TO 70
 IF a$ = "n" THEN GOTO 55
 ' ---------------------------------------------------------
' 2.1 Normal distributed variables with mean 0 and variance 1
 ri = 0
 FOR i = 1 TO 12
 cas = RND(3333): ri = ri + cas
 NEXT i
 ze1 = ri - 6
 ri = 0
 FOR i = 1 TO 12
 cas = RND(9999): ri = ri + cas
 NEXT i
 ze2 = ri - 6
 ' --------------------------------------------------------
' 2.2 Number of trees
 55 x = LOG(f + p)
 m = .024252 - .002357 * x - .008671 * y
 n = -.019991 + .001429 * x - .004050 * y
 zf=LOG(f):zp=LOG(p)
 e1=af - .04017 * zf + m * zp
 e2=ap - .04877 * zp + n * zf
 f2 = f * EXP(e1+.05*ze1)
 p2 = p *EXP(e2+.04*ze2)
 IF f2 > f THEN f2 = f * EXP(e1)
 IF p2 > p THEN p2 = p * EXP(e2)
 ' --------------------------------------------------------
' 2.3 Tree volumes
 vf=vff*.0007^EXP(-.041*(t-1)):vp=vpf*.0677^EXP(-.05*(t-1))
 ' --------------------------------------------------------
 ' 2.4 Standing volumes
 tvf=vf*f2:tvp=vp*p2
 ' --------------------------------------------------------
```

```
 ' 2.5 Printing decennial projected values
 IF INT (r%/10)<>r%/10 THEN GOTO 17
 LPRINT "Age: ";r%
 LPRINT "Quercus robur: ";fix(f2);" trees/ha     ";tvf;"       c.m./ha"
 LPRINT "Pinus pinaster: ";fix(p2);" trees/ha     ";tvp;"       c.m./ha"
 LPRINT
 17 f = f2: p = p2: r% = r% + 1
NEXT t
END
' --------------------------------------------------------
' --------------------------------------------------------
```

Appendix C. Sample of the output of simulator SB-PINHARDI

SIMUALTOR SB-PINHARDI
PROGRAM ROPIN

Age: 10
Quercus robur: 2500 trees/ha 8.75 c.m./ha
Pinus pinaster: 1200 trees/ha 103.08 c.m./ha

Age: 20
Quercus robur: 829 trees/ha 22.13144 c.m./ha
Pinus pinaster: 382 trees/ha 79.81063 c.m./ha

Age: 30
Quercus robur: 409 trees/ha 63.50119 c.m./ha
Pinus pinaster: 188 trees/ha 80.138886 c.m./ha