

The Stochastic Dynamics of Self-Thinned Pure Stands. A Simulative Quest

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Abstract. The author establishes and uses stochastic simulators for self-thinned even-aged, and uneven-aged pure stands of *Pinus pinaster* Ait. In both types of simulators he considers several levels of environmental stochasticity. From the application of the simulator for even-aged stands he concludes that the values in the range -1.45 to -1.49, found in the literature, and obtained from empirical data, do not allow the definitive rejection of the 3/2 power law. The simulator for uneven-aged stands has a mechanism to mimic different degrees of homeostasis. The author performed several sets of runs with this model, for combinations of values of environmental variability and of homeostasis. He then presents and interprets his simulated data. The model for uneven-aged stands reveals ergodicity. The author formally characterizes the dynamics of the area per tree.

Key words: area per tree; ergodicity; *Pinus pinaster* Ait.; self-thinned even-aged pure stands; self-thinned uneven-aged pure stands; stochastic simulators; 3/2 power law

A Dinâmica Estocástica para Povoamentos Puros Auto-Desbastados. Uma Inquirição Simulatória

Sumário. O autor estabelece e aplica simuladores estocásticos para povoamentos puros auto-desbastados de *Pinus pinaster* Ait., tanto regulares como irregulares. Em ambos os casos considera diferentes níveis de estocacidade ambiental. Da utilização do simulador para os povoamentos regulares ele conclui que provavelmente valores entre -1.45 e -1.49, obtidos a partir de dados empíricos, não permitem uma rejeição definitiva da lei dos 3/2. O modelo para os povoamentos irregulares tem um mecanismo para simular vários graus de homeostasis do povoamento. Para várias combinações de valores de variabilidade ambiental e homeostasis ele empreendeu grupos de simulações, cujos resultados apresenta e interpreta. O modelo para os povoamentos irregulares revela ergodicidade. O autor faz a caracterização formal da dinâmica da área por árvore.

Palavras-chave: área por árvore; ergodicidade; lei da potência dos 3/2; *Pinus pinaster* Ait; povoamentos puros auto-desbastados regulares; povoamentos puros auto-desbastados irregulares; simuladores estocásticos

La Dynamique Aléatoire pour Peuplements Purs Auto-Eclaircis. Une Enquête avec Simulations

Résumé. L'auteur établit et applique des simulateurs stochastiques pour les peuplements purs et auto-éclaircis de *Pinus pinaster* Ait., réguliers et irréguliers. Il conclue que probablement, les

valeurs de -1.45 à 1.48, obtenues à partir des données empiriques ne permettent pas le rejet définitif de la loi de l'exposant 3/2. Les simulateurs pour les peuplements irréguliers contiennent un mécanisme d'homéostasie contrôlable. L'auteur présente et interprète les résultats de différentes simulations avec divers niveaux stochastiques de l'environnement et d'homéostasie. Le modèle pour les peuplements irréguliers est ergodique. L'auteur fait une analyse formelle de la surface par arbre.

Mots clés: loi de l'exposant 3/2 ; peuplements purs et auto-éclaircis; *Pinus pinaster* Ait; simulateur ergodique ; simulateur stochastique ; surface par arbre

Introduction

The theory I established for self-thinned stands is a deterministic archetype of the presumed regularities evinced by the growth and structure of the same stands. The theory has the pretension to describe the way the self-thinned stands would grow if their developments were deterministic, and not stochastic, as it occurs in nature.

This deterministic conceptualisation accepts the following:

- Self-thinned stands are complex entities, but there is simplicity that emerges from this intricacy. This situation allows the description of self-thinned stands by simple equations (COHEN and STEWART, 1994).

- The growth of individuals obeys to eco-physiological, chemical and physical constrictions.

- The occupancy of the space by the population of trees of a self-thinned stands abides physical constraints.

The main alterations that occur in self-thinned stands are caused by the growth of the individuals, and the induced reduction of their number. Tree growth obeys to general laws that affect the dynamics of their density. The former regularities can be represented by allometric equations. Relationships between the relative growth rates of the variables that measure the size of the

individuals must exist. Also these relative growth rates must be related to the relative mortality rate (BARRETO, 1995, 2002b).

In a broader perspective, my theory converges to the main conceptions of James H. Brown, Brian J. Enquist, and Geoffrey B. West, that are explained, and illustrated in BROWN and WEST (2000).

Given the previous explanation, it is justified to require that my theory be submitted to the test implicit in the following question: *can the theory underpin a coherent stochastic simulation of the same stands?*

In this paper, I will attempt to satisfy this requirement in the case of self-thinned even-aged pure stands (SEPS), and self-thinned uneven-aged pure stands (SUPS).

To attain this scope:

- a) I must elaborate a conjecture of the mechanism of growth and self-thinning;

- b) Add stochasticity to it;

- c) Try to mimic the whole process with a simulator.

The main purpose of this paper is to disclose my accomplishment of this undertaking.

In my simulations, I will use SEPS and SUPS of *Pinus pinaster* Ait. (maritime pine, MP). MP is a pioneer species with fast initial growth, and short longevity, in the Mediterranean Basin, that occupies about forty per cent of the afforested area

of Portugal.

My model for environmental stochasticity is of the type independent and identical distributed sequences.

This paper is a revised, and enlarged version of BARRETO (2002c).

The stochastic dynamics of SEPS

The process of self-thinning in SEPS

The mechanism of self-thinning in SEPS is conceptualized as follows:

a) The dbh of the mean tree, in a small period of time, grows according to their Gompertzian pattern.

b) The increased size of the mean tree is incompatible with the actual density, because the mean tree occupies more space;

c) Self-thinning must actuate to reduce the area occupied by the trees to the area of the land where the stand is located (admitted to be one hectare, in this paper).

The way this mechanism actuates is described in part II of method SB-BARTHIN (BARRETO, 2001), and I will not repeat it, here.

To bring stochasticity to the previous process, I make the final or asymptotic value, in the Gompertz equation related to the dbh of the mean tree, a stochastic variable.

The basic procedure of simulation is the following one:

Generate a random number → generate a final value of dbh → simulate the dbh growth of the mean tree → increase the stem volume of the mean tree → adjust the density to the new size of the mean tree

The stochastic simulations

For my simulative purposes, I consider a SEPS of MP, and I admit that the final value of the dbh has a lognormal distribution. At age 10, this SEPS is characterized as follows: 1943 trees per hectare; mean dbh equal to 14.32 cm; the mean tree volume is 0.085684 m³.

I will present two deterministic simulations, and two stochastic simulations.

From here on, I will use the following notation: **p**= stand density, trees/ha; **d**=dbh of the mean tree, cm; **v**=stem volume of the mean tree, m³.

Simulation A. This deterministic simulation uses only the Gompertz equations (model KHABA, (BARRETO, 1990)) related to the forest variables.

Simulation B. It is also a deterministic simulation. For comparative purposes, it applies the mechanism described in the previous sub-section, to the deterministic final value of the **d**.

Simulation C. The final value of the dbh of the mean tree has the lognormal distribution (3.5582, 0.05).

Simulation D. The final value of the dbh of the mean tree has the lognormal distribution (3.5582, 0.15).

Now, I introduce pertinent information about the numerical procedures used in these four simulations.

In simulations A and B the final value of **d**, **d_f** is equal to 14.32/0.4076. In Simulation A, the final value of **p**, **p_f** is equal to 1432/6.018; and the final value of **v**, **v_f** is equal to 0.085684/0.0677.

All four simulations use the following equations:

$$d(t) = d_f \cdot 0.4076^{\exp(-0.05(t-10))} \quad (1)$$

$$v(t)=v_f 0.0677\exp(-0.05(t-10)) \quad (2)$$

Simulation A calculate $p(t)$ as follows:

$$p(t)=p_f 6.018\exp(-0.05(t-10)) \quad (3)$$

Simulation B, C, and D use the following algorithm:

Calculate the environmental quality associated to the random value of d_f :

$$Q=0.5679289 d_f + 4.039238 \quad (4)$$

Calculate the constant M :

$$M=0.000083781 Q^2 - 0.0009141 Q + 0.009827 \quad (5)$$

3 : Estimate v_f as follows:

$$v_f=M d_f \quad (6)$$

and after use eq. (2).

Now, the following equation is used to estimate the area occupied by the mean tree after growth (m^2 , BARRETO, 2001, eq. (9)):

$$A=10.427529 (d/20.38)^2 \quad (7)$$

The new density is given by:

$$p=10^4/A \quad (8)$$

The values associated to Simulations A-D are exhibited in table 1.

The values of b were obtained from the fitting (annual values) of the following equation:

$$\ln v=a+b \ln p \quad (9)$$

In table 1, I introduce a complete simulation of a SEPS, related to Simulations A, B, C, and D.

For each level of variability (C, D), I run 10 sets of 100 simulations. For each set, I calculated the mean of b , its variance, and the mean of the coefficient of determinations of the fitting of eq. (9). These figures are displayed in table 2.

For the case when the variation of d is characterized by lognormal (3.5582, 0.15), I also considered the following four situations:

Situation I. There is a propensity to the deterministic value of d_f to occur at

least 17 times, during the simulation.

Situation II. The value of d_f is not submitted to any restrictions.

Situation III. The value of d_f is equal or less to 37 cm.

Situation IV. Combines situations I, and III.

For each one of these situations I simulated 10 sets each one with 100 simulations. For each set I calculated the average value of b , and finally I estimated the average of these 10 values.

I found the following figures of b . Situation I: -1.4812; Situation II: -1.4822; Situation III: -1.4850; Situation IV: -1.4851.

In all 4000 simulations the coefficients of determination are greater then 0.9990.

Comments on the simulations

Let me introduce a few necessary notes about the previous simulations.

1) In table 1, simulations A and B are virtually identical. Thus the mechanism of self-thinning, and its associated simulation procedure are acceptable.

2) The values of b in simulations C and D, are consistent with the assumed variability of the final values of dbh. The value closer to -1.5 is associated to the less variable environment.

3) Referring to Situations I to IV, I verify that the level of propensity I choose has small impact in the deviation of the dynamics of the SEPS from the 3/2 power line.

4) The same Situations also suggest that, in real stands, it is probable that only the natural limitation on the variation of the environment is sufficient to confine the stand to a trajectory close to the 3/2 power line.

Table 1 - The figures associated to four simulations of a SEPS of *Pinus pinaster* Ait. To fit the values of **b**, annual values were used. For the meaning of symbols see the text

Age	Simulation A			Simulation B			Simulation C			Simulation D		
	p	d	v	p	d	v	p	d	v	p	d	v
10	1943	14.32	0.0857	1945	14.31	0.0855	1810	14.83	0.0946	1945	14.31	0.0859
20	959	20.38	0.2472	960	20.36	0.2473	760	22.88	0.3407	789	22.47	0.3200
30	625	25.25	0.4698	626	25.23	0.4702	586	26.07	0.5118	378	32.43	0.9410
40	482	28.75	0.6937	482	28.73	0.6943	424	30.63	0.8252	290	37.04	1.4061
50	412	31.10	0.8787	412	31.08	0.8793	363	33.14	1.0456	290	37.04	1.4061
60	374	32.63	1.0141	375	32.61	1.0149	315	35.57	1.2886	223	42.28	2.0849
70	353	33.59	1.1062	353	33.57	1.1071	279	37.80	1.5351	196	45.06	2.5123
80	341	34.18	1.1662	341	34.16	1.1670	279	37.80	1.5351	196	45.06	2.5123
b	-1.500			-1.500			-1.493			-1.484		
r²	1.000			1.000			1.000			1.000		

Table 2 - Values of **b** (eq. (9)) of ten sets of 100 runs of the simulator, for Simulations C, and D

Sets	Simulation C			Simulation D		
	Mean of b	Variance	Mean of r²	Mean of b	Variance	Mean of r²
1	-1.490087	1.592E-05	0.999915	-1.482343	3.987E-05	0.999860
2	-1.490409	1.989E-05	0.999910	-1.483298	9.269E-05	0.999854
3	-1.489134	1.613E-05	0.999912	-1.480690	5.984E-05	0.999850
4	-1.489928	1.688E-05	0.999915	-1.482284	5.223E-05	0.999864
5	-1.489920	1.688E-05	0.999918	-1.481596	5.912E-05	0.999860
6	-1.489925	1.574E-05	0.999911	-1.482684	6.001E-05	0.999868
7	-1.489414	1.769E-05	0.999919	-1.482252	5.477E-05	0.999869
8	-1.489964	1.702E-05	0.999919	-1.482818	5.435E-05	0.999870
9	-1.490255	2.116E-05	0.999919	-1.482680	6.185E-05	0.999851
10	-1.489255	1.223E-05	0.999919	-1.481254	4.837E-05	0.999879
Mean	-1.489829			-1.482190		

It is my understanding that the main conclusion I can infer from the simulations is the following one: *probably, the values of **b** in the range -1.45 to -1.49, found in the literature, and obtained from empirical data, are not conclusive for the definitive rejection of the 3/2 power law.*

Now I approach the stochastic dynamics of SUPS.

The stochastic dynamics of SUPS

The SUPS to be simulated

My assumptions about the uneven-aged stands of MP are the following ones (BARRETO, 2002a):

1. The population is structured in age classes with a range of 10 years.

2. Trees with age smaller than 10 years are considered regeneration, subject both to inter and intraspecific competition. Thus, the first age class has trees with age from 10 to 19 years.

3. Trees with 80 years or older, in average and poor sites, have already attained a state of decrepitude that can be ignored for modelling purposes.

4. I will consider my MP population structured in seven age classes, in the range 10 to 79 years.

To characterize the dynamics of the SUPS of MP (USMP), I will consider one time period of ten years.

During a period of 10 years, the trees of a given age class will be self-thinned or will move to the next class, if they

escape natural mortality.

To establish model of the SUPS, for each class, I must estimate the fraction of trees that die (**M**), transit to the next class (**T**).

In SUPS, each age class occupies the same area (BARRETO, 1989).

My estimations of these parameters are exhibited in table 3. Also, in the same table, I insert a stable age distribution, referred as trees per class per hectare (y_i , $i = 1, 2, \dots, 7$), in a poor site.

My simulations use a Leslie model (LM) supported by the previous parameters. The associated Leslie or transition matrix is the one (BARRETO, 2002a) of eq. (10).

$$B := \begin{pmatrix} .1981 & .3010 & .3800 & .45 & .45 & .45 & .40 \\ .5584 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .7042 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .8093 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .8794 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .9258 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .9551 & 0 \end{pmatrix} \quad (10)$$

Table 3 - Characteristic parameters of the uneven-aged stand of MP, a stable age distribution, and the mean dbh of the classes (d_i , cm). y_i as trees per ha. A_i is the mean area occupied by a tree, m². For the meaning of **T** and **M** see the text

Class	T	M	y_i	d_i	A_i
I	0.5584	0.4416	282	11.12	5.066
II	0.7042	0.2958	157	14.93	9.099
III	0.8093	0.1907	111	17.82	12.870
IV	0.8794	0.1206	90	19.81	15.873
V	0.9258	0.0742	79	21.12	18.083
VI	0.9551	0.0449	73	21.96	19.569
VII	0.9643	0.0357	70	22.48	20.408

My main reference for the next analysis is Caswell (2001). The eigenvalues of this matrix (**EVB**) are:

$$\text{EVB}=[1, 0.387\pm0.607i, -0.194\pm0.643i, -0.594\pm0.264i]' \quad (11)$$

The dumping ratio associated to matrix **B** is 1.39, and the mean generation time length is 40 years.

The fraction of trees of each age class, when the stable age distribution is attained (**FR**) can be obtained from the eigenvector associated to the eigenvalue 1, as follows:

$$\text{FR}=[0.327, 0.183, 0.129, 0.104, 0.092, 0.085, 0.081]' \quad (12)$$

The reproductive values of the classes are given by vector **RV**:

$$\text{RV}=[1, 1.437, 1.613, 1.523, 1.220, 0.832, 0.040]' \quad (13)$$

The reproductive values of the classes increase till class III, and after they decline. This is a common pattern in aged-structured biological populations.

The sensitivity of the dominant eigenvalue to the elements of matrix **B** (eq. (10)) are exhibited in the next matrix **S**, eq. (14).

The elasticities associated to matrix **S** are displayed in the following matrix **E**, eq. (15).

Matrix **S** (eq. 14) shows that the higher sensitivities are associated to the regeneration of the SUPS. This finding corroborates a truth already known by foresters. They are aware that the stand regeneration is a critical issue in the management of uneven-aged stands. This evidence will be used to establish my strategy for simulation.

$$\text{S} := \begin{pmatrix} 0.280 & 0.402 & 0.451 & 0.426 & 0.341 & 0.223 & 0.112 \\ 0.156 & \color{red}{\blacksquare} & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & 0.158 & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & 0.143 & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & 0.119 & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & 0.088 & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & 0.058 & \blacksquare \end{pmatrix} \quad (14)$$

$$\text{E} := \begin{pmatrix} 0.055 & 0.121 & 0.171 & 0.192 & 0.153 & 0.100 & 0.045 \\ 0.087 & \color{red}{\blacksquare} & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & 0.111 & \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & 0.116 & \blacksquare & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & 0.105 & \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & 0.081 & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare & \blacksquare & 0.055 & \blacksquare \end{pmatrix} \quad (15)$$

A strategy for simulation

My purpose is to show that my unified theory for self-thinned stands has the capability of supporting stochastic simulations of SUPS. I am not attempting to model any set of empirical data. Thus, without loss of generality I can choose any level of the effects of environmental stochasticity (ES) upon the dynamics of the SUPS.

I preferred simplicity. Given matrix **S** (eq. 14), I consider only the impact of ES upon regeneration, this is, its effect on the values of the first line of matrix **B** (eq. 10). Also, tentatively, I embedded a mechanism of homeostasis in the simulator. The mechanism is also simple, but is enough for my scope. The basic self-control of the SUPS is estimated by a variable **r**. When $r=1$, the SUPS behaves as if the environment is deterministic (complete homeostasis regarding to ES). This mechanism establishes the link between ES and the vital rates.

Let **s** be a random value of the total density of the SUPS. The variable **m** is calculated as:

$$m = -\ln r/s \quad (16)$$

At a given instant, **n** is the total density of the SUPS. The variable **c** is estimated as:

$$c = r \exp(m n) \quad (17)$$

The ES is introduced by multiplying the values of the first line of matrix **B**, by **c**.

Let me clarify the behaviour of **c**.

Case 1. $s < n$ implies $c < 1$. There is an improvement in the site quality, and the SUPS moves toward a structure with smaller total density, but larger trees.

Case 2. $s > n$ implies $c > 1$. There is a degradation of the site quality. The SUPS seeks a larger total density with smaller

trees.

Case 3. $s = n$ implies $c = 1$. The SUPS reacts as if the environment is deterministic.

Now, let me clarify the effect of **r**. Let me consider two values of **r**, **r**₁ and **r**₂. Also, $s_1 = s_2$, and $n_1 = n_2$. In case 1, if $r_1 > r_2$ then $c_1 < c_2$. In Case 2, if $r_1 > r_2$ then $c_1 > c_2$. The increasing of **r** amplifies the response of the SUPS in each direction.

I also considered that, in each class, there is an upper limit to the tree size.

I considered values of **r** equal to 1.1, 4.1, 7.1, 10.1, 13.1, 16.1, 19.1.

Now, I must consider the generation of random values of **s**.

Total density is a discrete variable. A value of 999.333 trees/ha is impossible for the density of a SUPS, but 999333 trees is an acceptable value if the same homogeneous forest spreads for thousand hectares. The area unit is an arbitrary choice.

Also, I could choose for each class of the SUPS structure a random mean value of the dbh (a continuous variable), and after calculate the frequencies of each class, and the total density. I tried this method and the behaviour and patterns of the simulations are the same, as for the algorithm I applied. I call this procedure algorithm 2, and I will describe it ahead. Also, I do not know any reason to reject the hypothesis of a random environment with continuous variation without constant rate of change.

First, tentatively, I used a discrete distribution for **s**, but I obtained a very coarse set of values. Thus, I choose to use a lognormal distribution for **s**, with mean equal to the total density of the stable structure (table 3), and variances 0.25, 0.45, 0.65, 0.85.

The main algorithm of the simulator is as follows:

Generate a random number \rightarrow generate a random value for $s \rightarrow$ calculate $m \rightarrow$ calculate $c \rightarrow$ multiply the figures in the first line of matrix B by $c \rightarrow$ calculate the new frequencies of the classes \rightarrow for each class, calculate the area occupied by the mean tree \rightarrow check, and adjust, if the upper limits of the classes are violated \rightarrow calculate the total area occupied by the new frequencies \rightarrow check if this area is greater than 10000 m^2 , and eventually adjust the frequencies \rightarrow use the new value of total density, and the previous one to calculate the natural logarithm of their ratio (growth rate)

Briefly, the SUPS detects the new environmental situation s , filters the impact with its mechanism of homeostasis, and after accommodates to the new conditions.

The value of s is assumed as the aggregate estimation of the average environmental conditions in the next decade, and this length of time is considered sufficient for the internalisation of the change by the SUPS.

Although I do not explicitly consider the effects of ES upon M , and T , it occurs in a coherent manner when exceeding

trees are self-thinned. Also, the size of the mean tree of each class is not immune to the effects of ES.

The algorithm of the simulator guarantees the harmonic impact of ES upon all relevant biological parameters of the tree population. The initial impact is upon the vital rates directly associated to regeneration, but, after, this effect coherently propagates to the remaining structural and dynamical aspects of the SUPS.

The simulations

For each pair $[r, \text{variance}]$, I simulated a set of 10 runs of the program, being the initial situation the stable structure. Each run performed 10^5 interactions. From each run, I retained the last values of total density (p , trees/ha), space effectively being used (AU , m^2), and the estimation of the mean stochastic growth rate (λ_s). The means of the sets are exhibited in table 4. These means are equivalent to use 10^6 interactions, this is, a period of 10^7 years.

Table 4 - Results of the stochastic simulations of the SUPS. For all combinations of r and variance, it is verified $\lambda_s = 0.999998$. For the meaning of the symbols see the text. Figures as follows: mean/standard deviation

Variance	Variables	r							
		1.1	4.1	7.1	10.1	13.1	16.1	19.1	Mean
0.25	p	723/0.02	714/0.10	711/0.08	710/0.07	709/0.13	708/0.13	708/0.13	712
	AU	9987/0.07	9942/0.51	9927/0.54	9918/0.48	9912/0.74	9908/0.78	9904/0.79	9928
0.45	p	719/0.04	696/0.28	689/0.16	685/0.25	684/0.29	682/0.25	681/0.19	691
	AU	9975/0.14	9707/2.35	9699/1.87	9696/2.66	9694/2.95	9693/2.32	9692/1.90	9737
0.65	p	711/0.15	680/0.54	673/0.18	668/0.45	666/0.31	664/0.19	663/0.26	675
	AU	9910/1.51	9475/3.63	9506/3.49	9519/4.12	9527/3.81	9532/3.30	9532/4.15	9572
0.85	p	686/0.54	669/0.54	662/0.23	657/0.34	654/0.43	652/0.38	651/0.39	662
	AU	9587/7.03	9319/4.89	9383/4.05	9408/5.61	9422/4.73	9428/4.60	9434/4.25	9426
Mean	p	709	690	684	680	678	677	676	-
	AU	9864	9610	9629	9635	9639	9640	9641	-

The figures in table 4 let me formulate the following assertions:

A1. All values of total density are smaller than the one of the stable structure (862).

A2. Total density asymptotically declines with increasing values of r . Probably, overreaction is not an advantageous property.

A3. Total density decreases with increasing environmental variability. This does not surprise.

A4. There is a trend to the area being used to decrease with increasing values of r .

A5. The area being used decreases with increasing environmental variability.

A6. The mean stochastic growth rate, λ_s , converges to the dominant eigenvalue of matrix \mathbf{B} , this is, to 1 (eq. (11)).

Statements A1-A5 are coherent and consistent.

Statement A6 is not an uncommon occurrence.

Ergodicity

An interesting property of stochastic simulators is ergodicity. For more details about it see chapter 14 in CASWELL (2001).

In regard to this property, I register two verifications:

V1. If I run my simulator, with any initial structure, with the same set of random numbers, for 10^5 interactions, I obtain always the same structure.

V2. If I extend the number of interactions to 10^6 , I obtain a structure very close to the stable one (table 3).

The observed ergodicity of my

simulator has an interesting biological meaning. The vital rates embedded in matrix \mathbf{B} are capable of determining the present age structure regardless of the structure of the SUPS in the past. This occurrence confers, at least, formal verisimilitude to matrix \mathbf{B} , and to the structure and performance of my simulator. Also, this result grants credibility to the concept of a given forest structure associated to a site, and to a level of soil use (tree spacing), this is, to a given index of performance (BARRETO, 1995).

Description of algorithm 2

Algorithm 2 can be described as follows:

1. Find a random value of a lognormal distribution of d_1 (table 3).
2. Calculate $d_2=1.342626 d_1$,
 $d_3=1.602518 d_1$, $d_4=1.781475 d_1$,
 $d_5=1.899280 d_1$, $d_6=1.974820 d_1$,
 $d_7=2.021853 d_1$
3. Calculate $A_i=0.0409678 d_i^2$.
4. Calculate $y_i=1428.57142/ A_i$.
5. Calculate $s=\sum y_i$.

The dynamics of the area occupied by the tree

The dynamics of the area occupied by the tree (\mathbf{A}) can be easily characterized. First, remember that this variable has a linear dimension with power 2. Now, I apply the method described in BARRETO (1995) and the information of table 3, exhibited in the same article.

Let \mathbf{z} represents the variables with linear dimension (dbh, heights, volume per area unit). For a variable \mathbf{y} , $R_y=y(10)/y_t$, $y(10)=$ value of \mathbf{y} at age 10.

Then, it can be written:

$$RA = R_p^{-1} \quad (18)$$

$$RA = R_z^2 \quad (19)$$

$$RA = R_v^{2/3} \quad (20)$$

Let **RGR_y** represent the relative growth rate of **y**, and **RMR** the relative mortality rate, at a given age. Thus, from the previous equations, I obtain:

$$RGRA = -RMR \quad (21)$$

$$RGRA = 2 RGR_z \quad (22)$$

$$RGRA = 2/3 RGR_v \quad (23)$$

I can also write:

$$A(t) = A(10) R_p^{(1 - \exp(-c(t-10)))} \quad (24)$$

The Gompertzian model for the area per tree is:

$$A(t) = A_f RA \exp(-c(t-10)) \quad (25)$$

For instance, for MP, $RA = 6.018^{-1} = 0.166$.

Eqs. (21)-(23) imply the other fundamental relationships of the SEPS growth:

$$RGR_z = -\frac{1}{2} RMR \quad (26)$$

$$RGR_v = 3 RGR_z \quad (27)$$

$$RGR_v = -3/2 RMR \quad (28)$$

$$R_z = R_p^{-0.5} \quad (29)$$

$$R_v = R_z^3 \quad (30)$$

$$R_v = R_p^{-3/2} \quad (31)$$

Final comments

I already punctuated my text with specific comments pertinent to the issues I approached. Thus, here, I only state a few general remarks.

My main conclusion is that my unified theory for self-thinned stands successfully satisfied the test mentioned in the *Introduction*: I developed coherent and consistent stochastic simulators both for SEPS, and SUPS.

The procedure described in part II of

method SB-BARTHIN (BARRETO, 2001) creates the possibility of several alternatives of simulation for the stochastic modelling of self-thinned pure stands. I illustrated this potential in this paper.

My simulations of SEPS bring a new insight to the controversial interpretation of the empirical data related to the 3/2 power law.

The literature dealing with SUPS is scarce, particular under a stochastic perspective. This situation may reinforce the interest of my results in this area.

I illustrated that a simple model can generate coherent and consistent data. This data and the family of models I here start introducing can contribute to the clarification of relevant issues in forest ecology, and foster the formulation of new conjectures and their simulated evaluation.

This is a very positive attribute for a scientific theory.

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