The Changing Geometry of Self-Thinned Mixed Stands. A Simulative Quest

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Abstract. The author uses an analytical procedure he developed, and simulated data of self-thinned even-aged mixed stands of *Quercus robur+Pinus pinaster, Pseudotsuga menziesii+Picea sitchensis,* and *Pinus nigra* ssp.*laricio+Pinus pinaster,* to scrutinize the allometry and fractal geometry of this type of stands. He verified that they are highly variable. They continuously change with initial proportions of each population, and age.
Key words: allometry; fractal geometry; self-thinned mixed stands; *Picea sitchensis; Pinus nigra* ssp.*laricio; Pinus pinaster; Pseudotsuga menziesii; Quercus robur*

A Geometria Variável dos Povoamentos Regulares Auto-Desbastados Mistos. Uma Inquirição Simulatória

Sumário. O autor usa um procedimento analítico que desenvolve e simulações de povoamentos auto-desbastados regulares mistos de *Quercus robur+Pinus pinaster, Pseudotsuga menziesii+Picea sitchensis,* e *Pinus nigra* ssp.*laricio+Pinus pinaster,* para propor uma clarificação da alometria e geometria fractal deste tipo de povoamentos. Verifica-se que estas são altamente instáveis, variando com as proporções inicas das populações e idade.
Palavras-chave: alometria; fractais; povoamentos auto-desbastados mistos; *Picea sitchensis; Pinus nigra* ssp. *laricio; Pinus pinaster; Pseudotsuga menziesii; Quercus robur*

Le Changement de Géométrie des Peuplements Réguliers Auto-Éclaircis. Une Enquête avec Simulation

Résumé. L'auteur simule des peuplements auto-éclaircis réguliers mixtes de *Quercus robur+Pinus pinaster, Pseudotsuga menziesii+Picea sitchensis,* et *Pinus nigra* ssp.*laricio+Pinus pinaster*. Il utilise les résultats de ces simulations pour clarifier l'allométrie, et la géométrie fractale de ces mêmes peuplements. Sa conclusion est que toute la géométrie des peuplements mixtes change avec les proportions initiales des espèces et avec l'âge.
Mots clés: allométrie ; fractale ; peuplements auto-éclaircis réguliers mixtes ; *Picea sitchensis; Pinus nigra* ssp. *laricio; Pinus pinaster; Pseudotsuga menziesii; Quercus robur*

Introduction

This paper is a follow up of BARRETO (1995), in which I deal with the allome- try, fractal geometry, and scaling factors of self-thinned even-aged pure stands (SEPS). Now, I will attempt to extend the same analysis to self-thinned even-aged

mixed stands (SEMS), using the same simulative approach. The present effort is also related to my clarification of the 3/2 power law (BARRETO, 1994a), my concept of tree competition (BARRETO, 1997, 1999b), and to my typification of the patterns of tree interaction (BARRETO, 1999a). Here, I will not repeat the concepts and findings I presented in these papers.

In the present article, I will show that, in SEMS, the allometry, fractal geometry, and scaling factors vary with the proportions of trees of each species, and the ratio of their relative growth rates, this is, they change during the life of the stand.

From here on, referring to a tree population in a SEMS, the variable $b(t)$ is the power in the following equation:

$$v(t)\, Pm(t)^{b(t)} = K \qquad (1)$$

$v(t)$ is the mean tree stem volume at age t, $Pm(t)$ is the number of trees per area unit at the same age, and K is a constant that varies with age.

Also, this paper enlarges my seminal result that $b(t)$, in SEMS, vary with age, stated in BARRETO (1990, table 7). This verification was illustrated also in BARRETO (1997, 2000a).

$b(t)$ is related to the relative growth of v $(RGRv(t))$, and the relative mortality rate $(RMR(t))$, at age t, as follows (BARRETO, 1997, eq. (12)):

$$b(t) = -RGRv(t)/RMR(t) \qquad (2)$$

I suggest the explicit introduction of age (t), in eqs. (11)-(13) presented in BARRETO (1997), for more rigor and clarity.

The development of this paper can be described as follows:

a) First, I will describe my simulative and analytical methods, and show how the value of $b(t)$ is related to the main allometry and fractal geometry of a tree population in SEMS.

b) I will use simulations of SEMS of *Quercus robur*+*Pinus pinaster* (QR+PP), and of *Pseudotsuga menziesii*+*Picea sitchensis* (PM+PS), to illustrate the variation of $b(t)$..

c) I will apply the results derived in a) to simulated SEMS of *Pinus ningra* ssp. *laricio*+*Pinus pinaster* (PN+PP). These species are used because I am able to simulate the dynamics of the biomass of their populations (BARRETO, 1994, 1999c, 2000b, 2004).

d) I close the paper with adequate final comments.

In all simulations, my model BACO2 was used.

This article is a revised version of BARRETO (2002b).

The simulative approach

In this section, I will describe the method I used in my analysis.

I simulated the standing biomass of crown, stem and roots, of PN and PP, in SEMS with initial proportions of PN equal to 0.2, 05, 0.8. For ages 20,30, … 90, I calculated the relative growth rates, and the relative mortality rates for the two populations.

I assumed that these rates are associated to instantaneous Gompertzian patterns of variation, and I calculated the correspondent ratios of "value at age 10/final value".

To the SEMS QR+PP, and PM+PS, I applied the analytical procedure described in the next section.

The analytical procedure

Now, I will describe the analytical approach I also used.

First, I must have the simulated data

of the number of trees of each population, in SEMS, for several pairs of consecutive years. These changes of the number of trees allow me to calculate the relative mortality rates, of the populations at several ages. I apply these values to eq. (2), to obtain the values of *b(t)*.

To obtain the key value of the power of the linear dimension, for the number of trees per area unit, $x_p(t)$, I use the following equation:

$$x_p(t) = -3/b(t) \qquad (3)$$

To estimate the same power for other population variables, I only have to algebraically add the value of $x_p(t)$ to the correspondent (constant) value for the tree (BARRETO, 1995, table 3).

For a given population variable, to evaluate its ratio "value at age 10/final value" I rise the same ratio for the growth of a variable of the tree with linear dimension (e.g., dbh) to the correspondent power calculated as described in the previous paragraph. From this ratio, assuming an instantaneous Gompertzian growth, I can calculate the instantaneous relative rate of growth of the variable, at age *t*.

To check the accuracy of this method, I compared the values of the relative rates of variation it generates, with those estimated from the simulated values of two contiguous years, and I verified that they are virtually the same for the mortality rate, and also very close for the relative growth rates of the biomasses of the crown, stem, roots, and total standing tree biomass of the populations.

The competitive hierarchies of the selected SEMS

In my conceptual model for tree competition, the Grime's hypothesis is accepted. Thus the ratio of the relative growth rates, of the species in competition, determines the competitive dominance. The species with larger relative growth rate is the dominant. In this context, it is relevant to show how the ratio of the relative growth rates behaves in the three SEMS I will analyze here. These ratios are exhibited in Table 1.

Referring to my typification of the interactions of tree populations (BARRETO, 1999a), I verify the following:

a) The interaction of the SEMS QR+PP is Type I: the ratio increases with age.

b) The interaction of the SEMS PN+PP is Type III, but very close to Type II: the ratio decreases with age, but is almost constant.

c) The interaction of the SEMS PM+PS is Type VI: the ratio is very close to 1, and a shift of dominance occurs early.

It is convenient to interpret the meaning of the values of *b(t)*. If b(t)>3/2, then the population is dominant, its (stem) standing volume is increasing faster then in pure stand, and it transfers to the dominated species some of its mortality (less intense self-thinning); the effect of the other population upon it is a positive coefficient of competition. If 1<b(t)<3/2, then the population is dominated, its standing volume is increasing slower then in pure stand, and suffers a more intense self-thinning; the effect of the other population upon it is a negative coefficient of competition. If b(t)=1, the difference with the previous situation is the actual constancy of the standing volume. If b(t)<1, now the standing volume decreases with age.

Now, I move to the analysis of the mixed stands of QR+PP, and PM+PS, where I only estimated the values of *b(t)*, as already stated.

The SEMS with QR+PP

In Table 2, I exhibit the values of *b(t)*, in three SEMS, when the initial proportion of the number of trees of QR, at age 10, is 0.2, 0.5, 0.8. The stands are the same used in BARRETO (1999a).

This table shows that the values of *b(t)* mirror, in an adequate manner, the competitive hierarchy, the effect of the proportion of the dominant species, and the coefficients of competition of this type of interaction, displayed in BARRETO (1999a, Table 1A). Also, the overall values of *b* reflect correctly the competitive hierarchy of the stand.

The SEMS with PM+PS

Here, I repeat the previous analysis to the SEMS of PM+PS. In Table 3, I exhibit the values of *b(t)*, in three SEMS, when the initial proportion of the number of trees of PM, at age 10, is 0.2, 0.5, 0.8. The stands are the same used in BARRETO (1999a).

The values of *b(t)*, in Table 3, has the same correct performance as those in table 2, and they reflect the general patterns of variation of the coefficients of competition of this type of interaction, displayed in BARRETO (1999a, Table 6A).

Table 1 - The ratios of the couple of species here considered

Ages	10	20	30	40	50	60	70	80	90
QR/PP	2.209	2.417	2.645	2.894	3.167	3.465	3.791	4.148	4.539
PN/PP	2.500	2.476	2.451	2.426	2.402	2.378	2.355	2.331	2.308
PM/PS	0.987	1.007	1.027	1.048	1.069	1.091	1.112	1.135	1.158

Table 2 - The values of *b(t)*, in SEMS of QR+PP, for three initial fractions of the trees of QR, at age 10 (fr). The last two lines of the table show the adjusted values of *b* (least squares regression of the logarithmic form), for the decade values of each population, and the associated coefficient of determination

Age	fr=0.2		fr=0.5		fr=0.8	
	QR	PP	QR	PP	QR	PP
10	4.3459	1.3916	2.6913	1.1669	1.9738	1.0077
20	4.6434	1.2893	2.7680	1.0869	2.0002	0.9461
30	5.0844	1.2157	2.9074	1.0328	2.0651	0.9030
40	5.7771	1.1519	3.0930	0.9868	2.1608	0.8621
50	6.6750	1.0996	3.3466	0.9500	2.2394	0.8649
60	7.9715	1.0672	3.5909	0.9149	2.3573	0.7318
70	11.0686	1.0173	3.9206	0.8126	2.4435	0.7145
80	14.4441	1.0080	4.2364	0.8607	2.6082	0.6735
90	28.5563	0.9682	4.7720	0.7817	2.6241	0.5869
b	5.0558	1.1832	2.8291	0.9985	1.9781	0.8667
R^2	0.989	0.999	0.997	0.999	0.998	0.999

Table 3 - The values of *b(t)*, in SEMS of PM+PS, for three initial fractions of the trees of QR, at age 10 (fr). The last two lines of the table show the adjusted values of *b* (least squares regression of the logarithmic form), for the decade values of each population, and the associated coefficient of determination

Age	fr=0.2		fr=0.5		fr=0.8	
	PM	**PS**	**PM**	**PS**	**PM**	**PS**
10	1.6834	1.7016	1.6890	1.7074	1.6946	1.7131
20	1.6334	1.6184	1.6299	1.6150	1.6263	1.6115
30	1.6221	1.5751	1.6090	1.5627	1.5961	1.5505
40	1.6253	1.5463	1.6021	1.5252	1.5795	1.5207
50	1.6379	1.5262	1.6040	1.4967	1.5715	1.4683
60	1.6565	1.5115	1.6114	1.4738	1.5688	1.4381
70	1.6795	1.5002	1.6226	1.4546	1.5693	1.4117
80	1.7050	1.4910	1.6364	1.4377	1.5724	1.3879
90	1.7355	1.4560	1.6514	1.4224	1.5769	1.3659
b	1.5392	1.4949	1.5268	1.4834	1.5145	1.4721
R^2	1.000	1.000	1.000	1.000	1.000	1.000

Also, the overall values of *b* mirror correctly the competitive hierarchy of the mixture. As the two populations have very close relative growth rates the values of *b(t)* are about 1.5.

The SEMS with PN+PP

The values of b(t)

To have an adequate understanding of the behavior of the populations of PN and PP, in SEMS, the only possible paradigm is the characteristic parameters of their dynamics in pure stands. Let me write the Gompertz equation, for a general forest variable, as:

$$y(t) = y_f Ry^{exp(-c(t-10))} \qquad (4)$$

where y_f is the final value of *y*, $Ry = y(10)/y_f$. For a given species, *c*, and *Ry* are constant.

In table 4, I introduce the values of *c* and *Ry*, for SEPS of PN, and PP. In Table 5, I exhibit the power of the linear dimension, associated to the same variables. This table is equivalent to Table 3, in BARRETO (1995).

The main allometry, and the fractal geometry, of the SEPS of these two species, for any independent variable, can be obtained from Table 5, as described in BARRETO (1995).

In Table 6, I introduce the values of *b(t)*, for simulated SEMS of these two Mediterranean pines, when the initial fraction of trees of PN is 0.2, 0.5, 0.8. As the ratio of the relative growth rates of the two species is almost constant, the values of *b(t)* are mainly controlled by the proportions of the trees of the two competitors. As in the previous SEMS already analyzed, the lower is the proportion of the dominant species, the higher are the figures of *b(t)*. In SEMS with fr=0.2 and fr=0.5, the proportion of PN, increases respectively, form age 10 to 90, from 0.20 to 0.33, and from 0.50 to 0.53, thus *b(t)* decreases, with age. When fr=0.8, the same proportion decreases from 0.80 to 0.72, and *b(t)* increases with age.

Table 4 - The values of **c**, and *Ry* for PN and PP, in SEPS

Variable	PN	PP
c	0.051	0.050
Stand density	81.433	6.018
Tree dbh, and height, stem volume or biomass per area unit	0.1108	0.4076
Tree crown biomass	0.0122	0.1656
Tree stem volume, and biomass	0.0014	0.0677
Tree root biomass	0.0016	0.0745
Root biomass per area unit	0.1208	0.4498
Total biomass per area unit	0.1732	0.5040

Table 5 - Power of the linear dimension of several variables, in SEPS of PN and PP

Variable	PN	PP
Stand density	-2	-2
Tree dbh, and height, stem volume or biomass per area unit	1	1
Tree crown biomass	2	2
Tree stem volume, and biomass	3	3
Tree root biomass	2.948	2.911
Tree total biomass	2.933	2.914
Crown biomass per area unit	0	0
Root biomass per area unit	0.948	0.911
Total biomass per area unit	0.933	0.914

Table 6 - The values of *b(t)*, in SEMS of PN+PP, for three initial fractions of the trees of PN, at age 10 (fr). The last two lines of the table show the adjusted values of *b* (least squares regression of the logarithmic form), for the decade values of each population, and the associated coefficient of determination

Age	fr=0.2		fr=0.5		fr=0.8	
	PN	PP	PN	PP	PN	PP
10	5.8554	1.3638	3.0029	1.1220	2.0595	0.9568
20	4.6965	1.2724	2.8110	1.0779	2.0461	0.9402
30	4.2123	1.2300	2.7162	1.0605	2.0311	0.9368
40	3.9369	1.2074	2.6543	1.0521	2.0239	0.9363
50	3.7672	1.1957	2.6102	1.0483	2.0156	0.9374
60	3.6476	1.1903	2.5778	1.0480	2.0082	0.9392
70	3.5554	1.1878	2.5505	1.0489	1.9981	0.9402
80'	3.4991	1.1889	2.5286	1.0523	1.9936	0.9427
90	3.4265	1.1896	2.4959	1.0584	1.9833	0.9413
b	4.4383	1.2093	2.6573	1.0222	1.9083	0.8882
R2	0.997	1.000	1.000	1.000	1.000	1.000

The powers of the linear dimension

Now, I can use the figures in Table 6 to obtain the powers of the linear dimension, for the same stands, and ages, as displayed in Table 7. The powers of the tree variables are invariant, thus I only exhibit the figures for the stand variables.

Table 7 shows that the allometry and fractal geometry of each population, in a SEMS, change with age, during the stand life. Each age, has its unique stand geometry. It does not make sense to speak about the SEMS geometry, without mention not only the initial proportions of the species, but also the stand age. During its life, a SEMS does not evince a geometry, it has geometries.

Table 7 - Power of the linear dimension associated to stand variables of the SEMS of PN+PP. p=number of trees per area unit

Fr=0.2										
	PN					PP				
Age	P	Crown	Stem	Roots	Total	P	Crown	Stem	Roots	Total
10	-0.5123	1.4876	2.4876	2.4356	2.4206	-2.1996	-0.1996	0.8004	0.7113	0.7143
20	-0.6387	1.3612	2.3612	2.3092	2.2942	-2.3578	-0.3578	0.6422	0.5532	0.5562
30	-0.7122	1.2878	2.2878	2.2358	2.2208	-2.4389	-0.4389	0.5611	0.4720	0.4751
40	-0.7620	1.2380	2.2380	2.1860	2.1710	-2.4846	-0.4846	0.5154	0.4263	0.4294
50	-0.7963	1.2036	2.2036	2.1516	2.1366	-2.5089	-0.5089	0.4911	0.4021	0.4051
60	-0.8224	1.1775	2.1775	2.1255	2.1105	-2.5204	-0.5204	0.4796	0.3906	0.3935
70	-0.8438	1.1562	2.1562	2.1042	2.0892	-2.5257	-0.6257	0.4743	0.3852	0.3882
80	-0.8572	1.4283	2.1428	2.0908	2.0758	-2.5234	-0.5234	0.4766	0.3876	0.3906
90	-0.8755	1.1245	2.1244	2.0725	2.0575	-2.5219	-0.5219	0.4781	0.3891	0.4921
fr=0.5										
10	-0.9990	1.0009	2.0009	1.9489	1.9339	-2.6737	-0.6737	0.3256	0.2373	0.2402
20	-1.0672	0.9328	1.9328	1.8808	1.8658	-2.7832	-0.7832	0.2167	0.1277	0.1307
30	-1.1044	0.8955	1.8955	1.8435	1.8285	-2.8288	-0.8288	0.1712	0.0822	0.0852
40	-1.1302	0.8698	1.8698	1.8178	1.8028	-2.8514	-0.8514	0.1486	0.0596	0.0626
50	-1.1493	0.8507	1.8507	1.7987	1.7837	-2.8619	-0.8619	0.1381	0.0491	0.0521
60	-1.1638	0.8362	1.8362	1.7842	1.7692	-2.8625	-0.8625	0.1374	0.0484	0.0514
70	-1.1762	0.8237	1.8237	1.7717	1.7567	-2.8601	-0.8601	0.1399	0.0509	0.0539
80	-1.1864	0.8135	1.8135	1.7615	1.7465	-2.8508	-0.8508	0.1491	0.0601	0.0631
90	-1.2020	0.7980	1.7980	1.7460	1.7310	-2.8346	-0.8344	0.1654	0.0764	0.0794
Fr=0.8										
10	-1.4566	0.5433	1.5433	1.4913	1.4763	-3.1355	-1.1355	-0.1355	-0.2241	-0.2215
20	-1.4662	0.5338	1.5338	1.4818	1.4668	-3.1909	-1.1909	-0.1909	-0.2799	-0.2769
30	-1.4770	0.5230	1.5230	1.4710	1.4560	-3.2024	-1.2024	-0.2024	-0.2914	-0.2884
40	-1.4822	0.5177	1.5177	1.4657	1.4507	-3.2039	-1.2039	-0.2039	-0.2930	-0.2900
50	-1.4884	0.5116	1.5116	1.4596	1.4446	-3.2003	-1.2003	-0.2003	-0.2863	-0.2863
60	-1.4938	0.5061	1.5061	1.4541	1.4391	-3.1943	-1.1943	-0.1943	-0.2833	-0.2803
70	-1.5014	0.4986	1.4986	1.4466	1.4316	-3.1909	-1.1909	-0.1909	-0.2800	-0.2769
80	-1.5048	0.4952	1.4952	1.4432	1.4282	-2.1822	-1.1822	-0.1822	-0.2712	-0.2682
90	-1.5126	0.4873	1.4874	1.4353	1.4204	-2.1871	-1.1871	-0.1871	-0.2761	-0.2731

In Table 7, when *b(t)* increases, the population needs less area to grow as much as in pure stand. The value of space for the population changes with *b(t)*. If b(t)>1.5 the power of the linear dimension of trees per area unit *(p)* is greater then –2. If b(t)<1.5 the area unit shortens and the power is smaller then –2. In both cases the power can become fractional.

The powers of the linear dimension for the variables of the population biomasses, change in the same sense of *b(t)* and reflect the new "values" of space. Linear dimensions smaller then the correspondent value of table 5 reflect a growth slower then in pure stand and even a decline of the biomass, if the power is negative.

The relative rates of mortality, and growth

Let me introduce a few new symbols. *Rp* is the ratio *Ry* referred to the number of trees per area unit. *RMR(t)* refers to relative mortality rate and *RGR(t)* to relative growth rate, both at age *t*. In Table 8, I introduce the figures for these new variables, for the simulated SEMS.

Some comments can be elaborated on table 8. Obviously, when Ry>1 the variable is decreasing with age and has a negative value of *RGR*, and the opposite occurs when Ry<1.

For the population of PN, the following is verified:

1. *Rp* is always smaller then 81.433, what is consistent with its dominance, and diminished *RMR*.

2. For the same age, the *RMR* increases from fr=0.2 to fr=0.8.

3. For the same age, the *RGR* decreases from fr=0.2 to fr=0.8.

4. The decline of the biomass of any component, with age, never occurs.

The performance of the population of PP evinces the following aspects:

1. *Rp* is always greater then 6.018, because its mortality is increased by the presence of PN.

2. For the same age, the *RMR* increases from fr=0.2 to fr=0.8, and the opposite happens with the *RGR*.

3. The decline of the crown biomass occurs in the three SEMS. The power of the linear dimension of the tree crown biomass is the smallest (2).

4. The decline of the biomasses of the roots and total biomass is verified only when fr=0.8. The power of the linear dimension of these two tree biomasses is between 2 and 3.

5. The decline of the biomass of the stem happens only when fr=0.8. This tree component has the largest power of the linear dimension (3).

The figures in tables 6,7,8 do not evince discrepancies. This verification supports the internal correctness, and coherence of the conceptual model and analytical procedure I used.

The relationships among the tree geometry, *b(t)*, and the dynamics of a population of a given species, in SEMS, are clarified as follows:

1. If b(t)<1.5 the crown biomass of the population declines.

2. If b(t)<1 the stem biomass of the population declines.

3. If b(t)<3/(power of the linear dimension of the biomass of the tree root) the root biomass of the population decline.

4. If b(t)<3/(power of the linear dimension of the biomass of the total tree) the total biomass of the population decline.

Table 8 - The values of *Rp* and *Ry*, for the simulated SEMS of PN+PP. In each age, in the second line, the values of *RMR(t)* and *RGR(t)* are exhibited.

						fr=0.2				
	PN					**PP**				
Age	**Trees/ha Rp**	**Crown Ry**	**Stem Ry**	**Roots Ry**	**Total Ry**	**Trees/ha Rp**	**Crown Ry**	**Stem Ry**	**Roots Ry**	**Total Ry**
10	3.0852	0.0334	0.0024	0.0029	0.0078	7.1946	1.3798	0.5740	0.6367	0.7894
	-0.0575	0.1733	0.3067	0.2981	0.2498	-0.0987	-0.0161	0.0278	0.02930	0.0118
20	4.0517	0.0478	0.0043	0.0050	0.0076	8.2914	1.5105	0.6254	0.6881	0.8084
	-0.0429	0.0932	0.1669	0.1623	0.1492	-0.0642	-0.0125	0.0142	0.0113	0.0064
30	4.7889	0.0596	0.0060	0.0070	0.0087	8.9172	1.5843	0.6567	0.7221	0.8120
	-0.0288	0.0519	0.0939	0.0913	0.0872	-0.0403	-0.0085	0.0077	0.0060	0.0036
40	5.3413	0.0681	0.0074	0.0085	0.0097	9.2889	1.6235	0.6730	0.7398	0.8243
	-0.0185	0.0297	0.0542	0.0527	0.0512	-0.0249	-0.0054	0.0044	0.0034	0.0021
50	5.7599	0.0744	0.0084	0.0096	0.0105	9.4922	1.6424	0.6809	0.7484	0.8249
	-0.0116	0.0172	0.0317	0.0308	0.0302	-0.0152	-0.0033	0.0026	0.0020	0.0013
60	6.0914	0.0792	0.0091	0.0104	0.0112	9.5909	1.6489	0.6834	0.7512	0.8227
	-0.0072	0.0101	0.0187	0.0182	0.0179	-0.0093	-0.0020	0.0016	0.0017	0.0008
70	6.3694	0.0830	0.0097	0.0110	0.0117	9.6261	1.6484	0.6832	0.7508	0.8193
	-0.0044	0.0059	0.0111	0.0108	0.0106	-0.0056	-0.0012	0.0009	0.0007	0.0005
80	6.6166	0.0862	0.0101	0.0115	0.0122	9.6235	1.7692	0.7211	0.7811	0.8145
	-0.0027	0.0035	0.0066	0.0064	0.0063	-0.0034	-0.0009	0.0005	0.0004	0.0003
90	6.8467	0.0893	0.0105	0.0120	0.0126	9.5988	1.6361	0.6778	0.7450	0.8100
	-0.0021	0.0019	0.0039	0.0038	0.0038	-0.0021	-0.0004	0.0003	0.0003	0.0002
						fr=0.5				
	Rp	**Ry**	**Ry**	**Ry**	**Ry**	**Rp**	**Ry**	**Ry**	**Ry**	**Ry**
10	8.9968	0.1267	0.0108	0.0126	0.0309	11.0080	2.1951	0.9323	1.0236	1.2726
	-0.1121	0.1054	0.2310	0.2229	0.1773	-0.1200	-0.0393	0.0036	-0.0017	-0.0120
20	10.4520	0.1410	0.0136	0.0158	0.0240	12.1447	2.2596	0.9459	1.0394	1.2188
	-0.0719	0.0602	0.1315	0.1270	0.1143	-0.0757	-0.0247	0.0017	-0.0012	-0.0060
30	11.3464	0.1518	0.0160	0.0184	0.0230	12.6492	2.2736	0.9480	1.0417	1.1812
	-0.0447	0.0347	0.0761	0.0735	0.0694	-0.0467	-0.0151	0.0010	-0.0007	-0.0031
40	12.0045	0.1594	0.0176	0.0202	0.0232	12.9107	2.2714	0.9447	1.0381	1.1562
	-0.0274	0.0203	0.0446	0.0430	0.0416	-0.0285	-0.0091	0.0006	-0.0004	-0.0016
50	12.5140	0.1654	0.0188	0.0216	0.0237	13.0198	2.2614	0.9393	1.0322	1.1374
	-0.0168	0.0119	0.0263	0.0254	0.0248	-0.0173	-0.0055	0.0004	-0.0002	-0.0009
60	12.9328	0.1703	0.0197	0.0225	0.0242	13.0361	2.2462	0.9321	1.0243	1.1218
	-0.0102	0.0070	0.0156	0.0151	0.0148	-0.0105	-0.0033	0.0003	-0.0001	-0.0005
70	13.2975	0.1746	0.0204	0.0233	0.0248	12.9964	2.2285	0.9242	1.0157	1.1082
	-0.0062	0.0042	0.0093	0.0090	0.0088	-0.0064	-0.0020	0.0002	-0.0000	-0.0003
80	13.6313	0.1784	0.0210	0.0239	0.0253	12.9239	2.2081	0.9153	1.0057	1.0950
	-0.0037	0.0025	0.0055	0.0054	0.0053	-0.0039	-0.0012	0.0001	-0.0000	-0.0001
90	13.9474	0.1824	0.0215	0.0246	0.0258	12.8326	2.1884	0.9068	0.9966	1.0836
	-0.0022	0.0015	0.0033	0.0032	0.0032	-0.0023	-0.0007	0.0001	-0.0000	-0.0001

Table 8 – Cont.

	fr=0.8								
	Rp	**Ry**	**Ry**	**Ry**	**Ry**	**Rp**	**Ry**	**Ry**	**Ry**	**Ry**
10	24.6097	0.4437	0.0436	0.0507	0.1175	16.6582	3.4503	1.4953	1.6381	2.0261
	-0.1634	0.0414	0.1598	0.1521	0.1093	-0.1407	-0.0619	-0.0201	-0.0247	-0.0353
20	25.5134	0.3908	0.0405	0.0468	0.0700	17.5106	3.3244	1.4063	1.5435	1.8064
	-0.0992	0.0288	0.0982	0.0938	0.0815	-0.0869	-0.0364	-0.0103	-0.0132	-0.0179
30	25.7391	0.3687	0.0403	0.0463	0.0574	17.6856	3.2144	1.3479	1.4802	1.6779
	-0.0598	0.0183	0.0591	0.0565	0.0526	-0.0529	-0.0225	-0.0055	-0.0072	-0.0095
40	26.0320	0.3591	0.0405	0.0465	0.0531	17.7151	3.1364	1.3087	1.4376	1.6004
	-0.0360	0.0113	0.0354	0.0339	0.0324	-0.0321	-0.0127	-0.0030	-0.0040	-0.0052
50	26.3575	0.3560	0.0410	0.0469	0.0515	17.6500	3.0769	1.2804	1.4067	1.5498
	-0.0217	0.0068	0.0212	0.0203	0.0197	-0.0194	-0.0076	-0.0017	-0.0023	-0.0030
60	26.6959	0.3560	0.0415	0.0474	0.0504	17.5251	3.0262	1.2572	1.3813	1.5126
	-0.0131	0.0041	0.0127	0.0121	0.0119	-0.0118	-0.0045	-0.0009	-0.0013	-0.0017
70	27.0387	0.3578	0.0420	0.0480	0.0509	17.3640	2.9813	1.2373	1.3595	1.4833
	-0.0079	0.0025	0.0076	0.0073	0.0071	-0.0071	-0.0027	-0.0005	-0.0008	-0.0010
80	27.3841	0.3600	0.0424	0.0484	0.0511	17.1813	2.9378	1.2181	1.3385	1.4573
	-0.0047	0.0015	0.0045	0.0043	0.0043	-0.0043	-0.0016	-0.0003	-0.0004	-0.0006
90	27.7285	0.3636	0.0430	0.0490	0.0515	16.9869	2.8984	1.2013	1.3202	1.4353
	-0.0029	0.0009	0.0027	0.0026	0.0025	-0.0026	-0.0010	-0.0002	-0.0002	-0.0003

The figures in Tables 5, 6, and 8 illustrate these assertions.

Modular geometry, scaling factors, and fractal structure

Before I move to more generic issues, I approach the following features of SEMS: modular geometry, scaling factors, and fractal structure. In a concise manner, I will attempt to extend to mixed stands the material of the pertinent sections, presented in BARRETO (1995).

I admit that in SEMS, as in SEPS, self-thinning is a neutral thinning. Thus, as SEPS, SEMS are self-similar entities, formed by modules. Similarly, if I hypothesize that, in SEMS, the final density of a module of each species is one tree, the number of trees, and area of each module is not constant, in a given stand, but changes instantaneously as **Rp**

changes with age (Table 8).

I use Table 7 to illustrate a case of the changing allometry of the SEPS of PN+PP.

Let me introduce the following allometric equation:

$$y=ad^m \tag{5}$$

where *d* is dbh, a variable with the power of the linear dimension equal to one.

If *y* is any variable considered in Table 7, the respective values of *m* are also the values of the power of the linear dimension exhibited in Table 7 (m=power of the linear dimension of *y*/1).

This procedure can be extended to other independent variables.

Also, the scaling factors that can be established between two different populations of the same species, in distinct SEMS, change with age.

The figures in Table 7, and the concept of instantaneous Gompertzian

pattern of variation of the population variables, in SEMS (Table 8), suggests that in a given instant, I can obtain the variable fractal geometry of SEMS, as it can be done for a SEPS of a given species, as I illustrated for PP, in BARRETO (1995).

Mixture of maximum standing biomass

One interesting question is the mixture that produces the highest total standing biomass of the tree populations, at a given age. I attempt an inquiry in this issue, and I select the age of 100 years, consistent with the short longevity of PP. My results are shown in table 9.

The maximum production occurs for fr=0.18. For the sake of completeness, in table 10, I introduce the simulated values of the total standing biomass in the SEMS with fr=0.2, 0.5,0.8.

Gompertzian simulators

In SEMS with competitors with very close *RGR*, Gompertzian models can give an acceptable approximation of the population dynamics, as I already showed (BARRETO, 1998a, 1999b, 2002a). The same happens when the ratio of the *RGR* is constant, or has little variation. I take advantage of this circumstance to propose Gompertzian simulators for the stands with fr=0.2, 05, 0.8, in Table 11.

The values used to fit the Gompertz equations were those of ages 20,30, …100. Thus, in table 11, the values of *Ry* refer to the ratio "value at age 20/final value".

The reader can verify that the *RGR* associated to the values of Ry displayed in Table 11 are very close to the values exhibited in Table 8, when t ≥ 20 years.

Table 9 - Variation of the total standing biomass of the trees, at age 100, in SEMS PN+PP, for several values of the initial fraction of the trees of PN (fr). Tons of dry matter per hectare (TB)

fr	0.025	0.05	0.1	0.15	0.175	0.18	0.19	0.20	0.3	0.5	0.65	0.8
TB	351.3	392.1	436.5	453.5	455.9	456.0	455.9	445.5	435.1	352.5	263.7	197.0

Table 10 - The dynamics of the total standing biomass of the trees, in three SEMS of PN+PP. Tons of dry matter per hectare

Age	Fr=0.2			Fr=0.5			Fr=0.8		
	PN	PP	Total	PN	PP	Total	PN	PP	Total
20	20.3	132.8	153.2	32.5	69.3	101.8	33.5	23.2	54.7
30	61.6	139.6	201.3	78.0	66.3	144.3	64.1	20.3	82.4
40	119.7	143.7	263.4	133.5	64.8	198.3	97.1	18.8	115.9
50	177.9	146.2	324.1	184.7	64.0	248.7	125.4	18.1	143.5
60	225.3	147.7	473.0	224.5	63.6	288.1	146.6	17.7	164.3
70	259.4	148.6	408.0	252.4	63.3	315.7	161.0	17.4	178.4
80	282.1	149.2	431.3	270.7	63.2	333.9	170.4	17.3	187.7
90	296.6	149.6	446.2	282.3	63.1	345.4	176.2	17.2	193.4
100	305.6	149.9	455.5	289.4	63.1	352.5	179.2	17.2	197.0

Table 11 - Gompertz equations for the number of trees (p), and the biomass of needles (N), live branches (L), dead branches (D), stem wood (W), stem bark (B), roots (R), and total biomass (T), per area unit, in SEMS of PN+PP. For each component, comparing with the original simulated values with model BACO2, the values evince an error less then 1.0% but for total biomaas of PN, fr=0.8, that shows an error equal to 1.33%, at age 30.

		fr=0.2		fr=0.5		fr=0.8	
		PN	**PP**	**PN**	**PP**	**PN**	**PP**
p							
	c	0.0467	0.0494	0.0498	0.0502	0.0514	0.0508
	Ry	2.7247	3.9727	4.6214	4.9376	7.7026	6.0923
N							
	c	0.0538	0.0476	0.0526	0.0507	0.0496	0.0521
	Ry	0.1929	1.3340	0.3273	1.6580	0.5457	2.0459
L							
	c	0.0538	0.0475	0.0526	0.0506	0.0496	0.0510
	Ry	0.1928	1.3342	0.3273	1.6583	0.5456	2.0469
D							
	c	0.0538	0.0474	0.0526	0.0507	0.0482	0.0519
	Ry	0.1929	1.3342	0.3273	1.6584	0.5453	2.0477
W							
	c	0.0528	0.0530	0.0518	0.0411	0.0505	0.0591
	Ry	0.0517	0.7752	0.0891	0.9634	0.1485	1.1893
B							
	c	0.0525	0.0528	0.0517	0.0411	0.0505	0.0602
	Ry	0.0525	0.7750	0.0891	0.9634	0.1486	1.1885
R							
	c	0.0526	0.0539	0.0517	0.0705	0.0505	0.0564
	Ry	0.0569	0.8215	0.0966	1.0214	0.1610	1.2608
T							
	c	0.0514	0.0502	0.0505	0.0630	0.0489	0.0577
	Ry	0.0637	0.8842	0.1081	1.0994	0.1802	1.3565

Final comments

I admit I enlarged the insight, I previously disclosed, about the intricate issue of the structure and dynamics of SEMS. Underpinned by my unified theory for self-thinned stands, my results have internal coherence, and consistency, but, indeed, they need a lasting empirical study to be fully corroborated, although my understanding is that the results exhibited by PUETMANN, HIBBS, and HANN (1994) are in the same line of conceptualisation as the one here presented.

The main feature to retain, from the analysis I developed in this paper, is the following one: *it is probable that the geometry of SEMS is constantly changing during the life of the stand.* The allometry, the fractal geometry, and the scaling factors, in any moment, are ephemeral realities.

The figures in Tables 2, 3, and 6 sustain the assertion that, in SEMS, the relative values of *b(t)* are a good

indicator of the competitive hierarchy, and the variation of the values of *b(t)* also mirror effectively the type of interaction occurring between the species of the mixture.

If I accept that there is a time-space symmetry between SEMS and self-thinned uneven-aged mixed stands, the latter ones are a mixture of several geometries.

It is the described variability of mixed stands that renders their management a very difficult task.

The analytical procedure I used is applicable to a population both in pure and mixed stands. This must not surprise in a unified theory for self-thinned stands. The only difference is that in pure stands *b(t)* is always equal to 1.5.

The results I here presented also suggest that algorithm BRAFO (BARRETO, 1998b) must only be applied to SEMS with species of close competitiveness, or evincing constant (or with small variation) ratio of *RGR*.

At the present stage of my inquiry on self-thinned stands, the core of this paper is my best conjecture about the moving structure of mixed stands. The knowledge of a better one will render me happier and more sage.

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