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Verification of a Model of Stand Density Control Diagram for Austrian Black Pine Plantations in Bulgaria

Tatiana Stankova* and Tzvetan Zlatanov**
Researchers
*Department of Forest Genetics, Physiology and Plantations
**Department of Silviculture
Forest Research Institute of Bulgarian Academy of Sciences. 132, St. Kliment Ohridski Blvd., 1756 Sofia, BULGARIA

Abstract. The Stand Density Control Diagram (SDCD) is an average stand-level model which is primarily used to derive density control schedules by management objective. One of its applications is to evaluate stand growth parameters at any time of stand development. The main objective of the present investigation is to examine a model of SDCD estimated for Austrian black pine (*Pinus nigra* Arn.) plantations in Bulgaria with a representative validation data set and to illustrate its application for estimation of density, mean breast height diameter and stand stock.

Equivalent mean diameter curves proved to be very good predictors for all studied growth parameters, having smaller relative errors than the Equivalent height curves. A result of practical importance is the insignificant difference between the direct (experimental dominant height) and the indirect (predicted dominant height) way of parameter estimation. Among all six examined ways of determination of the stand stock the Equivalent mean diameter curves based on the experimental values of basal area and dominant height showed best performance. The proposed model of SDCD avoids some disadvantages of the existing growth and yield models and supplies a reliable method for estimation of the stand growth parameters.
Key words: growth parameters; mean-dominant height relationship; yield prediction models; stand-level models

Verificação de um Modelo de Diagrama de Controlo de Densidade do Povoamento para Pinheiro Larício (*Pinus nigra* Arn.) em Plantações na Bulgária

Sumário. O Diagrama de Controlo de Densidade do Povoamento (DCDP) é um modelo de povoamento cujo principal objectivo é desenvolver esquemas de controlo da densidade, de acordo com o objectivo de gestão. Uma das suas aplicações principais é a avaliação dos parâmetros de crescimento do povoamento em qualquer momento da sua vida. O presente estudo tem como objectivo principal testar um modelo de DCDP desenvolvido para plantações de *Pinus nigra* Arn. na Bulgária, por comparação com um conjunto representativo de dados de validação e ilustrar as possibilidades da sua aplicação na estimação da densidade, diâmetro médio à altura do peito e volume em pé.

As curvas equivalentes de diâmetro médio demonstraram ser boas preditoras para todos os

Corresponding Author E-mail: tatianastankova@yahoo.com

parâmetros de crescimento estudados, tendo apresentado erros relativos mais pequenos do que as curvas equivalentes de altura. Um resultado com interesse prático é a ausência de diferenças significativas entre os valores das estimativas dos parâmetros obtidos pelo método directo (medição da altura dominante) e pelo método indirecto (altura dominante estimada).

De todos os seis métodos testados para avaliação do volume em pé, o método das curvas equivalentes de diâmetro médio, baseadas na determinação experimental da área basal e da altura dominante apresentou a melhor prestação. O modelo proposto (DCDP) evita algumas desvantagens dos modelos de crescimento e produção existentes e fornece uma ferramenta fiável para a estimação dos parâmetros de crescimento do povoamento.
Palavras-chave: parâmetros de crescimento; altura média; altura dominante; modelos de produção; modelos de povoamento

Vérification et Preuve d'un Modèle de Diagramme de Commande de Densité de Peuplement pour les Plantations de Pin Noir en Bulgarie

Résumé. Le diagramme de commande de densité de peuplement (DCDP) est un modèle au niveau de plantation qui est principalement employé pour la planification des programmes de commande de la densité avec pour objectif le contentement de critères économiques précis. Une de ses applications principales est d'évaluer des paramètres de croissance des plantations à tout moment de leur développement. L'objectif principal de la recherche actuelle est d'examiner un modèle de DCDP estimé pour les plantations de pin noir (*Pinus nigra* Arn.) en Bulgarie avec un modèle représentatif de validation et d'illustrer sa possibilité d'évaluation de densité, de diamètre moyen à hauteur de poitrine et de réserve des plantations.

Les courbes moyennes de diamètre avérées équivalentes montrent les facteurs prédictifs très bons pour tous les paramètres étudiés de croissance, ayant de plus petites erreurs relatives que la courbe équivalente de taille. Un résultat d'importance pratique est la différence insignifiante entre la manière directe (taille dominante mesurée) et indirecte (taille dominante estimée) de l'évaluation de paramètre. Parmi chacune des six manières examinées de la détermination de la réserve des plantations les équivalentes courbes moyennes de diamètre basées sur les valeurs mesurées du secteur basique et de la taille dominante ont montré la meilleure exactitude. Le modèle proposé de DCDP permit l'évasion de quelques inconvénients des modèles existants de croissance et de productivité et propose une méthode fiable pour l'évaluation des paramètres de croissance des plantations.
Mots clés: paramètres de croissance; rapport signifié-dominant de taille; modèles de prévision de la réserve; modèles au niveau plantations

Introduction

Stand Density Control Diagram (SDCD) is an average stand-level model which graphically illustrates the relationships between yield, density and mortality throughout all stages of stand development and is primarily used to derive density control schedules by management objective (NEWTON, 1997). The SDCD is based on ecological rules and is an sufficiently accurate as a practical model applicable to simulation of the growth and density dynamics of even-aged pure natural or man-made forest stands under a broad range of growth conditions. The SDCD is applied in three directions. First, it is used to evaluate stand growth parameters (mean breast height diameter, dominant height, density, yield) at any time of the stand development. The second application of the SDCD is to simulate various thinning regimes from the establishment to final harvesting of a plantation and to estimate the total yield and the corresponding profit. The third direction for application of the SDCD is to determine the optimal initial density of forest plantations in accordance with the preferred management objective and thinning regime.

The main objective of the present investigation is to examine a model of SDCD estimated for Austrian black pine (*Pinus nigra* Arn.) plantations in Bulgaria (STANKOVA, 2005; STANKOVA and SHIBUYA, 2007) with a representative validation data set and to illustrate the first direction of its application.

Materials and methods

Evaluated model (SDCD) – data range and model development

Data collection took place in 122 temporary sample plots of rectangular or circular form and of different sizes (50-1000m^2) established in Austrian black pine plantations. The plots were chosen to cover the variety of sites, densities and growth stages of the Austrian black pine plantations mainly in the mountainous part of south-western Bulgaria at altitudes from 275 to 1350m a.s.l. and slopes from 0 to 39 degrees. Beside the personally recorded data, additional data from 323 plots in *P. nigra* plantations, either granted by other researchers (ALEXANDROV *et al.*, 2002; MARINOV, 1999, 2002, MARINOV *et al.*, 1997) or published elsewhere were included in the data set. Data of mean breast height diameters (dbh), mean and dominant heights, density, basal area and stand stock were used in the model development. Dominant heights for the published data were additionally determined using the established allometric relationship to the mean height for Austrian black pine plantations (STANKOVA *et al.*, 2006) and the stand stock was estimated using the volume tables for Austrian black pine plantations by NEDYALKOV *et al.* (1983).

The main elements of the SDCD are Equivalent height curves, Full density line, Natural thinning curves, Equivalent mean diameter curves and Yield index lines. Equivalent height curves describe the relationship between stand yield and density at a given growth stage, which is presented by the dominant height class. They are expressed by the reciprocal equation of the C-D effect (HAGIHARA, 1998):

$$y = \frac{\rho}{At\rho + B} \quad (1)$$

$$v = \frac{1}{At\rho + B} \quad \text{or} \quad \frac{1}{v} = At\rho + B \quad (2),$$

where y, ρ and v are yield, density and mean stem volume, respectively, and At and B are regression coefficients, depending on the dominant height class $\hat{H}$ (STANKOVA and SHIBUYA, 2003). Full density line is the upper boundary of the stand yield – density relationship and connects the points of density – maximum yield combinations. It is a power function (YODA *et al.*, 1963):

$$v = K\rho^{-\alpha} \text{ or } y = K\rho^{1-\alpha} \quad (3),$$

where K and α are constants. Based on the formulae by STANKOVA and SHIBUYA (2003), the slope of the self-thinning line α and the reciprocal value of the asymptotic stand density ε were determined and used to fix the full density line, described by Eq. 3, where the value of the intercept K was estimated using the value of ε (STANKOVA and SHIBUYA, 2003). Natural thinning curves describe the yield growth of stands of given initial densities with time-lapse, considering the process of self-thinning. The curves describing the process of natural thinning were determined according to the formula by SHIBUYA (1995):

$$v = K\rho^{-\alpha} - f \text{ or } y = K\rho^{1-\alpha} - f\rho \quad (4),$$

where parameter f is estimated for 23 arbitrary initial densities (N_0= 444–-40000/ha) using the formula:

$$f = \frac{K}{(N_0)^{\alpha}} \quad (5)$$

Equivalent mean diameter curves are trajectories on the SDCD, which connect yield-density combinations of stands having the same mean diameter. The equivalent mean diameter curves were built on the quadratic mean diameter and preliminary estimate of the stand form height (*HF*) is involved. Stand form height was determined for each stand by the formula:

$$HF = \frac{y}{G} \quad (6),$$

where G is the basal area (m²/ha), and then approximated on the dominant height by linear regression:

$$HF = k + lH_{dom,} \quad (7)$$

where H_{dom} is dominant height, and k and l are constants. The formula for the equivalent diameter curves was obtained after subsequent substitution:

$$y = \frac{HF\pi\rho D_g^2}{40000} \text{ or } y = D_g^2 \frac{\pi\rho}{40000}(k + l\hat{H}) \quad (8)$$

where D_g is the equivalent mean diameter class (cm). The equivalent mean diameter curves in this study were constructed for the range from 2 to 32 cm, by 2 cm intervals. Yield index is estimated as a ratio of the yield per hectare of a given stand to the yield of a stand on the Full density curve in the same dominant height class. On the double logarithmic scale, the Yield index curves are presented by lines parallel to the Full density curve, i.e. they are determined by the equation:

$$y = K'N^{1-\alpha} \quad (9),$$

where K' is a constant.

Run test of residuals was applied and the residual plots were examined for the fitted regressions to check for data deficiencies. The goodness of fit of the regressions was estimated through the coefficient of determination (R^2), significance of the estimated regression (*F*-test) and its coefficients (*t*-tests).

Verification of the model – validation data set and examination functions

Validation data set consists of 20 sample plots, 18 of which are established in the region of Oporski hills. Oporski hills are situated in south-western Bulgaria, 40 kilometres east from the capital Sofia and connect Sredna Gora mountain with the Central Balkan range. Most of the plantations are situated on slopes of southern component exposure at elevations of 700 to 850 m a.s.l. Two sample plots are established at the foot of the mountain Vitosha near Sofia on slope of eastern exposure at 650 m a.s.l. The studied Austrian black pine plantations are from 30 to 80 years of age. The forest sites are predominantly dry and the soil is relatively poor, of small to intermediate depth. The plots are established on slopes from 0 to 26 degrees and plot size varies from 100 to 1400m².

In each plot, breast height diameters of all trees, heights of 20% of the trees and density were measured. The heights of the quadratic mean diameter trees and of the maximum diameter trees were measured to estimate the mean and the dominant height, respectively. Density, basal area and yield per hectare horizontal area were calculated. The main parameters estimated, the corresponding SDCD functional elements tested for their precision in regard to these parameters as well as the formulae for the predicted values of the parameters (Eqs. 10 - 16) are listed in Table 1.

Table 1 - The main parameters estimated and the corresponding SDCD functional elements tested

Predicted Parameter	**Prediction Function**	**Formula[1]**
Dominant height	Linear function of mean height	$Hdom_{pred}=aHmean+b$ Eq.10
Density	Equivalent diameter curves	$\rho_{pred}=\frac{40000y_{exp}}{(\pi HFdbh_{exp})}$ Eq.11 $HF=f(Hdom)$
	Equivalent height curves	$\rho_{pred}=\frac{By_{exp}}{(1-yA_t)}$ Eq.12 $A_t=f(\hat{H});B=f(\hat{H})$
Mean breast height diameter	Equivalent diameter curves	$dbh_{pred}=200\sqrt{\frac{y_{exp}}{\pi HF\rho_{exp}}}$ Eq.13 $HF=f(Hdom)$
Yield	Equivalent height curves	$y_{pred}=\frac{\rho_{exp}}{(A_t\rho_{exp}+B)}$ Eq.14 $A_t=f(\hat{H});B=f(\hat{H})$
	Equivalent diameter curves through basal area	$y_{pred}=G_{exp}HF$ Eq.15 $HF=f(Hdom)$
	Equivalent diameter curves through mean dbh	$y_{pred}=\frac{\pi D^2_{g_{exp}}HF\rho_{exp}}{40000}$ Eq.16 $HF=f(Hdom)$

[1] – abbreviation **exp** in subscript denotes the experimentally determined values of the variable from the validation data set

Percentage of error for each tested parameter and function were estimated following the formula:

$$\%error = \frac{Var_{exp} - Var_{pred}}{Var_{exp}} 100\% \qquad (17)$$

where Var_{exp} stand for the values of the tested parameter determined from the validation data, while Var_{pred} are their corresponding values predicted by the model. Descriptive statistics for the error percentages and error distribution by percentiles (10, 25, 50, 75, 90%) were estimated. In addition, the validation data set was compared to the values of the growth parameters predicted by the Growth and yield tables for Austrian black pine plantations (TSAKOV, 1983) and for Austrian black pine natural stands (NEDYALKOV, 1983) and their error percentages were collated with those estimated for the tested model of SDCD.

Results and discussion

The Stand Density Control Diagram for Austrian black pine plantations was estimated and constructed in both graphical (Figure 1) and functional forms and all regression equations of the proposed modified model of SDCD were fitted to the data set with high degree of determination and proven statistical significance (STANKOVA, 2005; STANKOVA and SHIBUYA, 2007). The model characterizes the spatial and temporal dynamics of Austrian black pine plantations in Bulgaria at broad range of densities, forest sites and growth stages from 4 to 28-m dominant height class (Figure 1). The uppermost border of the maximum density – maximum yield combinations (self-thinning line) was fixed, with self-thinning exponent α=1.75, and the trajectories of natural thinning for 23 initial densities were determined (STANKOVA, 2005; STANKOVA and SHIBUYA, 2007).

The mean-dominant height relationship estimated for Austrian black pine plantations: *Hdom = 0.991Hmean + 1.256* (STANKOVA *et al.*, 2006) was tested for the mean height range 10.9 – 21.3m and the corresponding dominant height range 11.9 – 23m of the validation data set. The estimated errors varied from -5.0 to 8.9%, of mean error 1.9% which indicated high level of precision of the linear model tested (Figure 2B). A tendency to underestimation of the dominant height above 20m by the prediction function was shown for this data set (Figure 2A).

For each of the parameters density, yield and breast height diameter the predicted values were evaluated in two ways: directly through the experimental dominant height and indirectly – through predicted dominant height using the mean-dominant height relationship. The main reason was that the dominant height still does not have pronounced application in forest inventory and planning in Bulgaria. Mean height is still the main stand characteristic which is estimated, recorded and referred to in the forest management plans. Verification of satisfactory goodness of fit of the model of SDCD through predicted dominant height would allow accelerating of model application in the forestry practice.

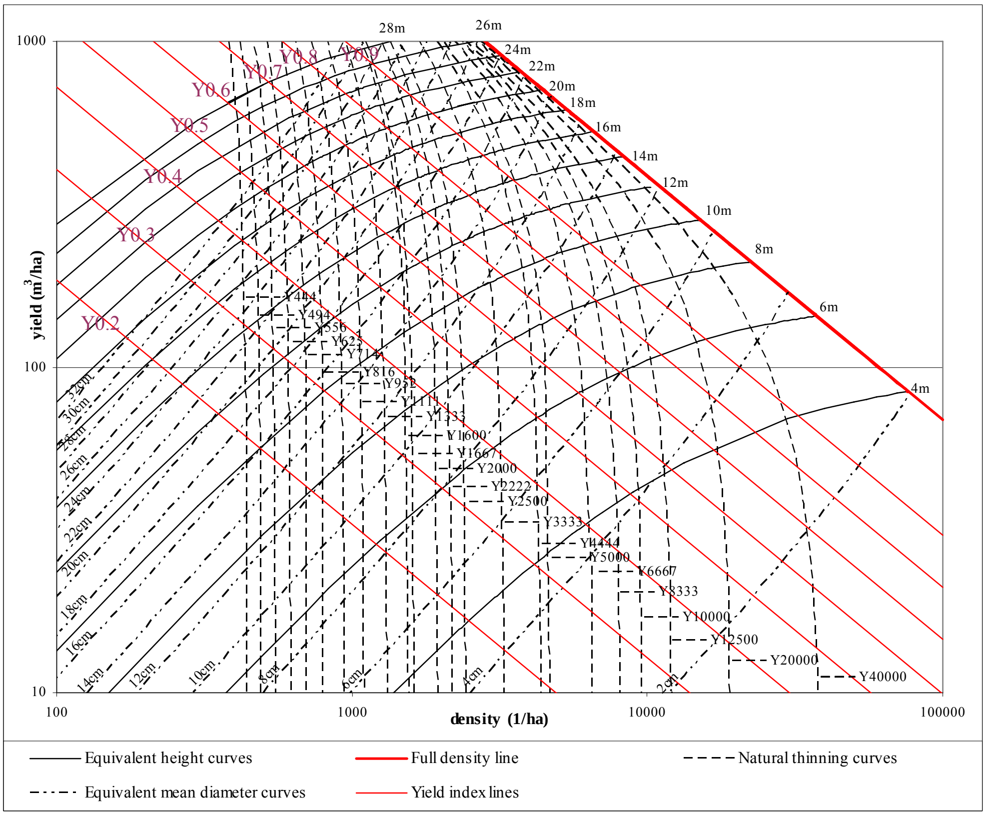
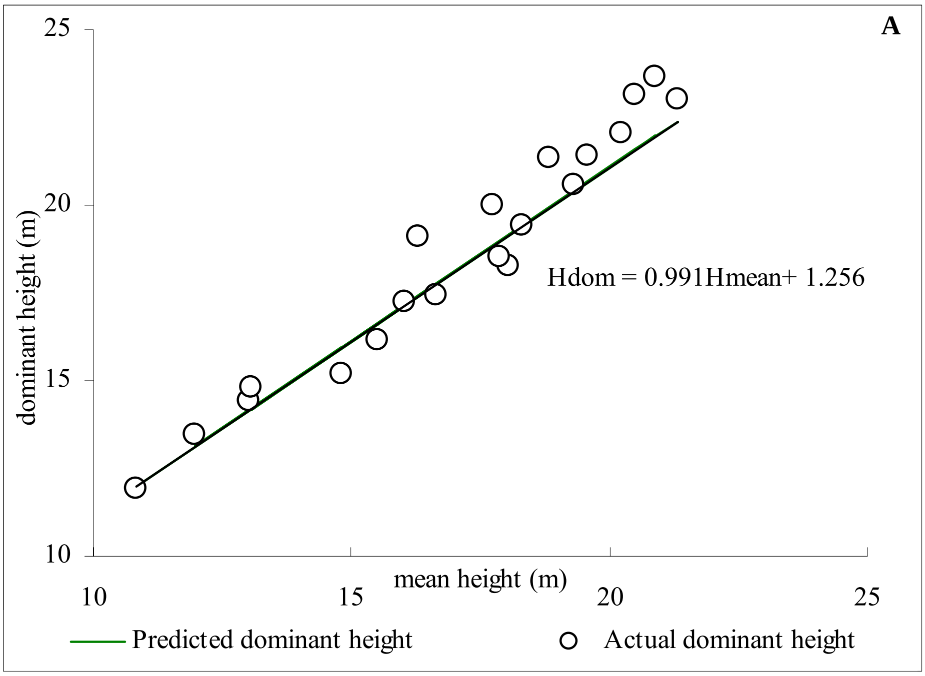


Figure 1 - Stand Density Control Diagram for Austrian black pine plantations in Bulgaria (STANKOVA, 2006)

The parameter stand density was estimated once through the Equivalent height curves (Eq. 12) and next – through the Equivalent diameter curves (Eq.11). Density estimation by means of Equivalent height curves (Figure 3) showed that the mean error values were -2.71% and 7.73% trough predicted and experimental dominant height, respectively. The error range was relatively large, indicating tendency to underestimation for the experimental dominant height predictor and to overestimation in case of the predicted dominant height. The values of the 10th and the 90th error percentiles were: (-68.49%; 36.47%) and (-35.57%; 51.56%) for predicted height and experimental dominant height, respectively. Much better results were obtained for the Equivalent diameter curves (Figure 4). The mean of the error percentages amounted to 0.10% and -1.99% for predicted and experimental dominant height cases, respectively.



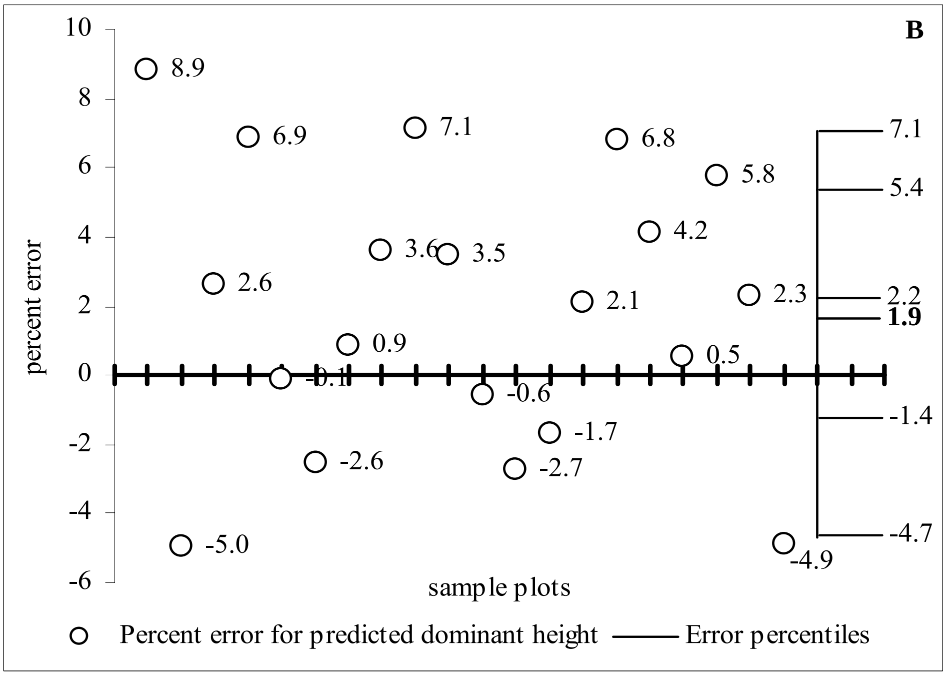


Figure 2 - Estimation of dominant height by the linear regression on the mean height: **A.** Actual dominant height vs. predicted through the mean height; **B.** Error percentages for predicted dominant height by sample plots (the values for 10th, 25th, 50th, 75th, 90th percentiles and the **mean** are indicated)

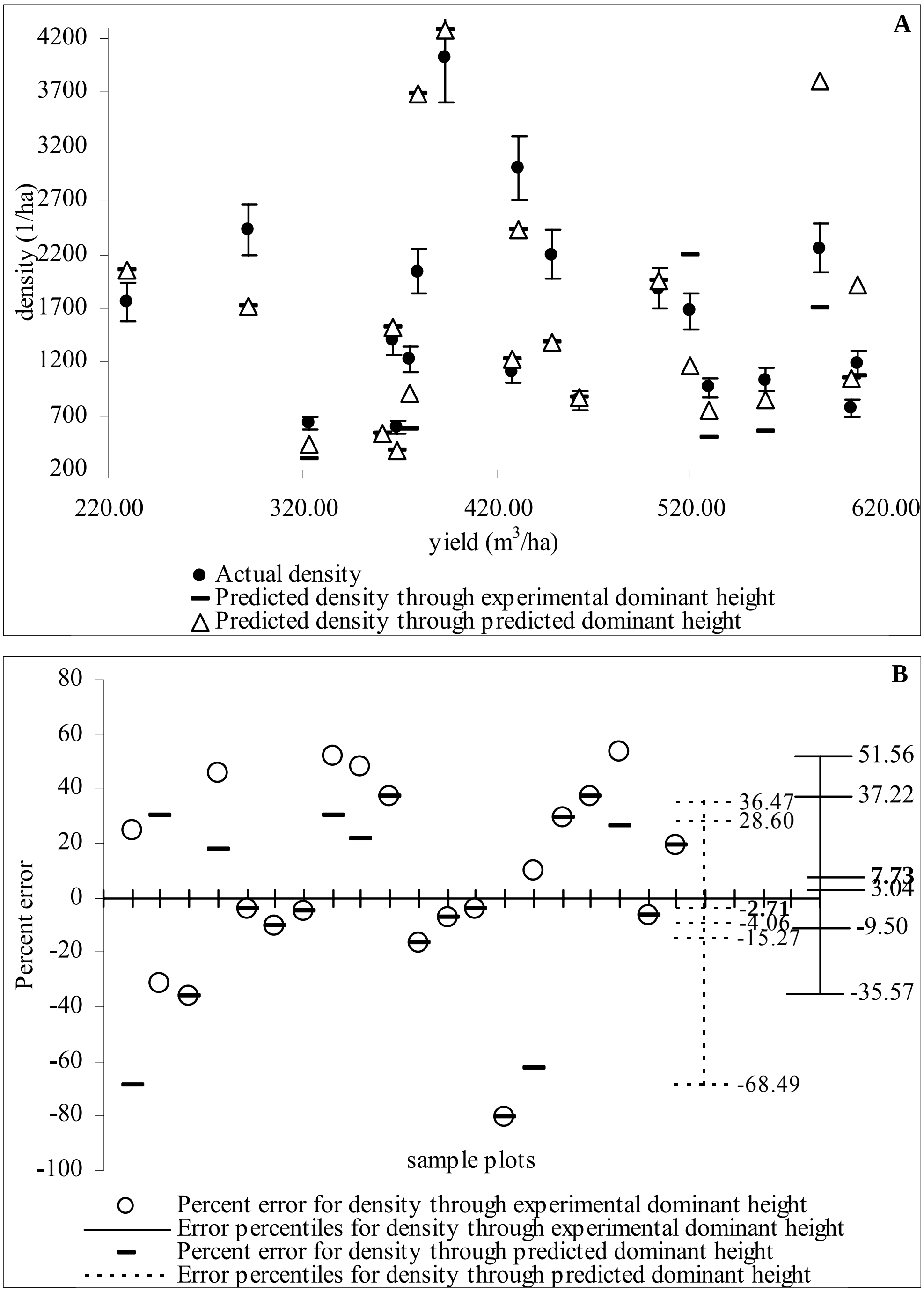


Figure 3 - Estimation of density by the Equivalent height curves: **A**. Actual density ±10% vs. predicted density by the Equivalent height curves; **B.** Error percentages for predicted density by sample plots (the values for 10th, 25th, 50th, 75th, 90th percentiles and the **mean** are indicated)

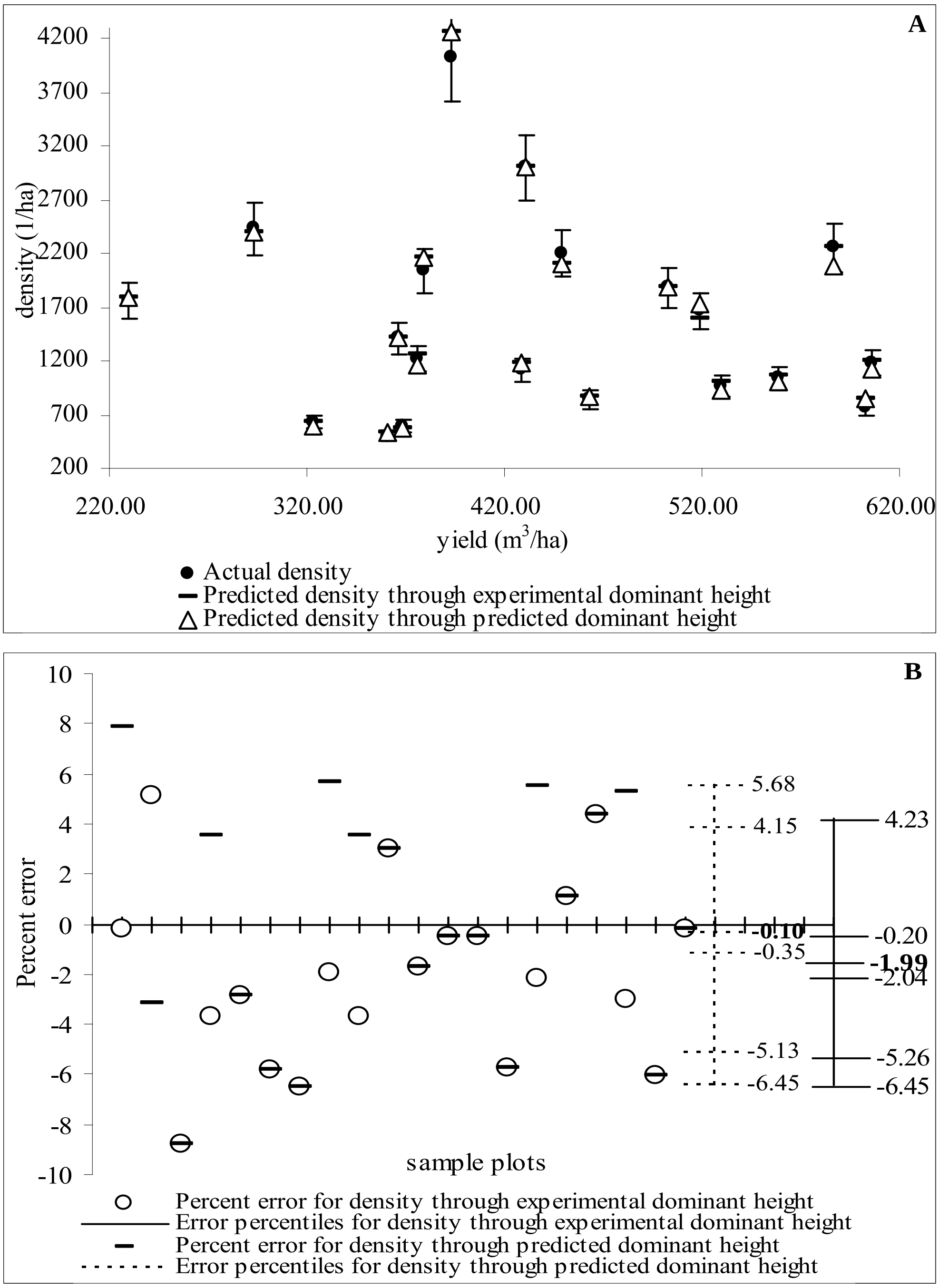


Figure 4 - Estimation of density by the Equivalent diameter curves: **A**. Actual density ±10% vs. predicted density by the Equivalent diameter curves; **B.** Error percentages for predicted density by sample plots (the values for 10th, 25th, 50th, 75th, 90th percentiles and the **mean** are indicated)

The predicted densities in all cases were in the ±10% range from the actual density and the indirect estimation of density through the predicted dominant height showed to be reliable enough in the case of Equivalent diameter curves. No evidence of density error dependency on the stand stock was observed for both Equivalent height curves and Equivalent diameter curves.

The parameter prediction ability of the Equivalent diameter curves was proven strong for the mean breast height diameter as well, regardless of the dominant height category (predicted or experimental) included. The absolute value of the mean percentage of error was less than 1% and 80% of all error values fell within a 6% range around the mean (Figure 5).

Stand stock, which is stand characterristic of major importance, was estimated in six different ways. Considering some practical methods for basal area determination which disregard the breast height diameter measurements, basal area along with the diameter was considered as an input variable when the Equivalent diameter curves were employed (Eqs. 15 and 16). The Equivalent height curves exhibited general tendency to parameter overevaluation, with lowering the value of the 10^{th} percentile to – 39.7% and -21.8% error for predicted and experimental dominant height, respectively. The Equivalent height curves showed better goodness of fit when the predicted value of the dominant height was involved, resulting in mean percentage of error -2.9% versus -8.4% for the experimental dominant height case (Figure 6). The Equivalent diameter curves proved to have better goodness of fit than the Equivalent height curves, concerning the stand stock as growth parameter. The mean error values varied from -0.12% to 2.1% and the absolute maximum error value was 16.1% (Table 2). The indirect yield prediction through the predicted dominant height did not differ considerably from the direct prediction method, having its error percentages more uniformly distributed around 0 (Figures 7 and 8). Stand stock estimation through basal area appeared to be the most precise, having 80% of the estimated errors in the range ±6% around the mean (Figures 7B and 8B).

Although the estimated relative error percentages are representative estimates of the goodness of fit of the examined model of SDCD, a comparison to the existing growth and yield models for Austrian black pine stands would allow better assessment of SDCD quality and practical applicability for stand growth parameters determination. Nowadays, the growth and yield tables for Austrian black pine plantations by TSAKOV (1983) and the growth and yield tables for Austrian black pine natural stands by NEDYALKOV (1983) are the officially published and employed in the practice models in Bulgaria. Plantations are classified in 4 categories and the natural stands – in 5 categories, according to their relative site index, which is defined in regard to their mean height and age class (by 5 or 10 years). Standing stock of each site index at full stocking rate (1.0) is characterized, in age classes from 5 (10) to 70 (140) years, by mean height, mean breast height diameter, density, basal area, stem form factor, yield, current and mean annual volume increment.

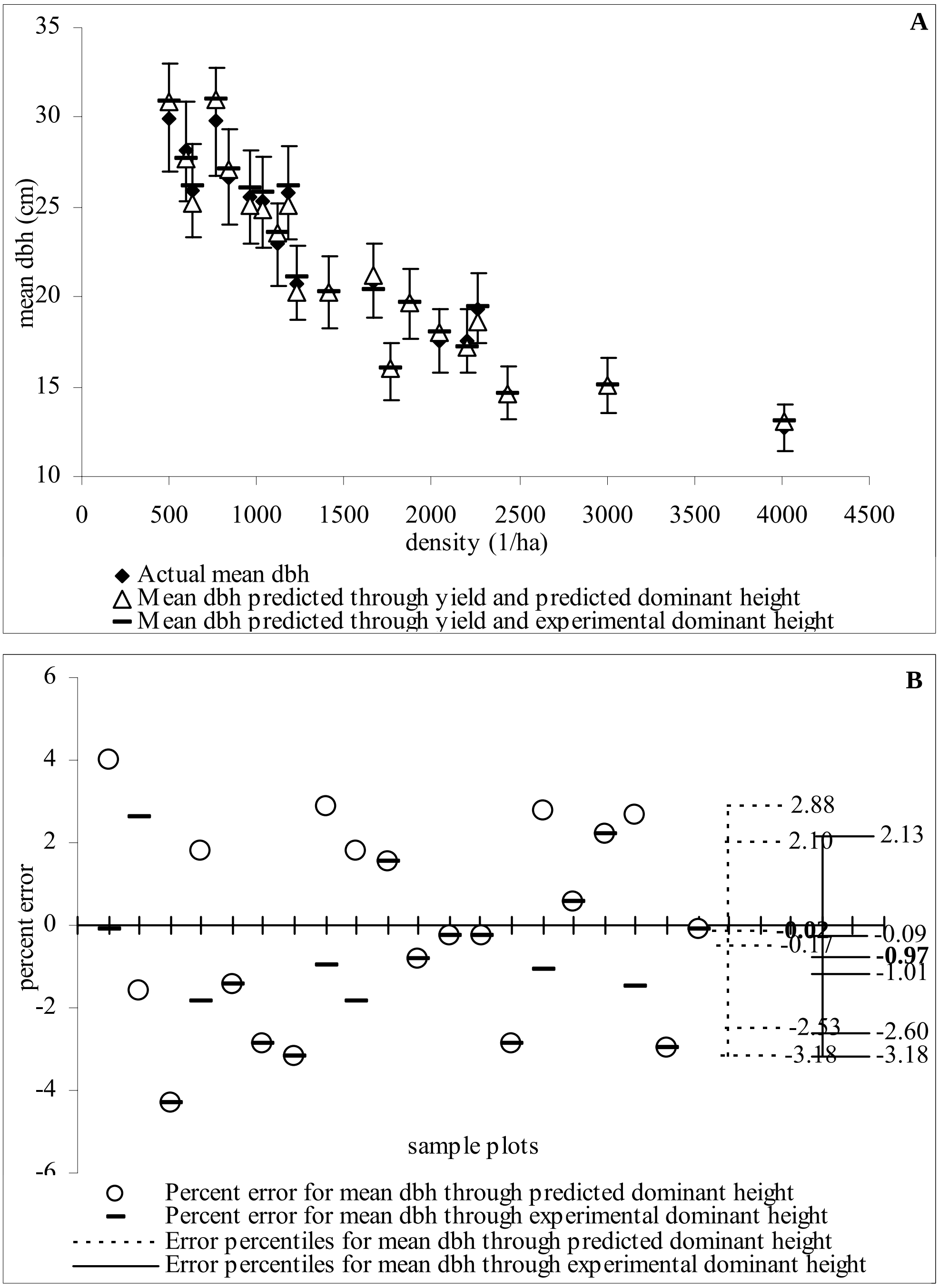


Figure 5 - Estimation of mean dbh by the Equivalent diameter curves: **A.** Actual mean dbh ±10% vs. predicted mean dbh by the Equivalent diameter curves; **B.** Error percentages for predicted mean dbh by sample plots (the values for 10th, 25th, 50th, 75th, 90th percentiles and the mean are indicated)

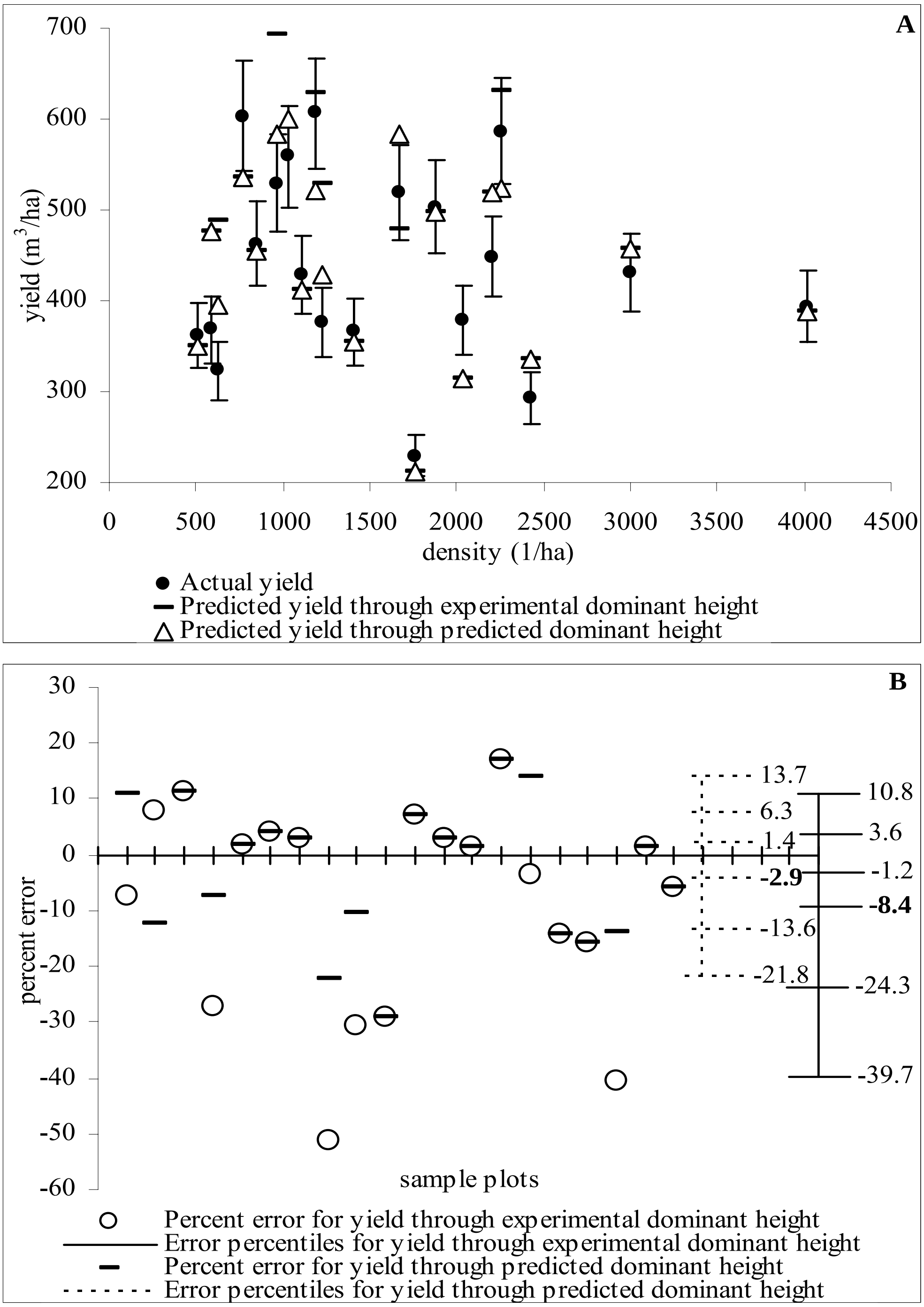


Figure 6 - Estimation of yield by Equivalent height curves: **A.** Actual yield ±10% vs. predicted by the Equivalent height curves; **B.** Error percentages for predicted yield by sample plots (the values for 10th, 25th, 50th, 75th, 90th percentiles and the **mean** are indicated)

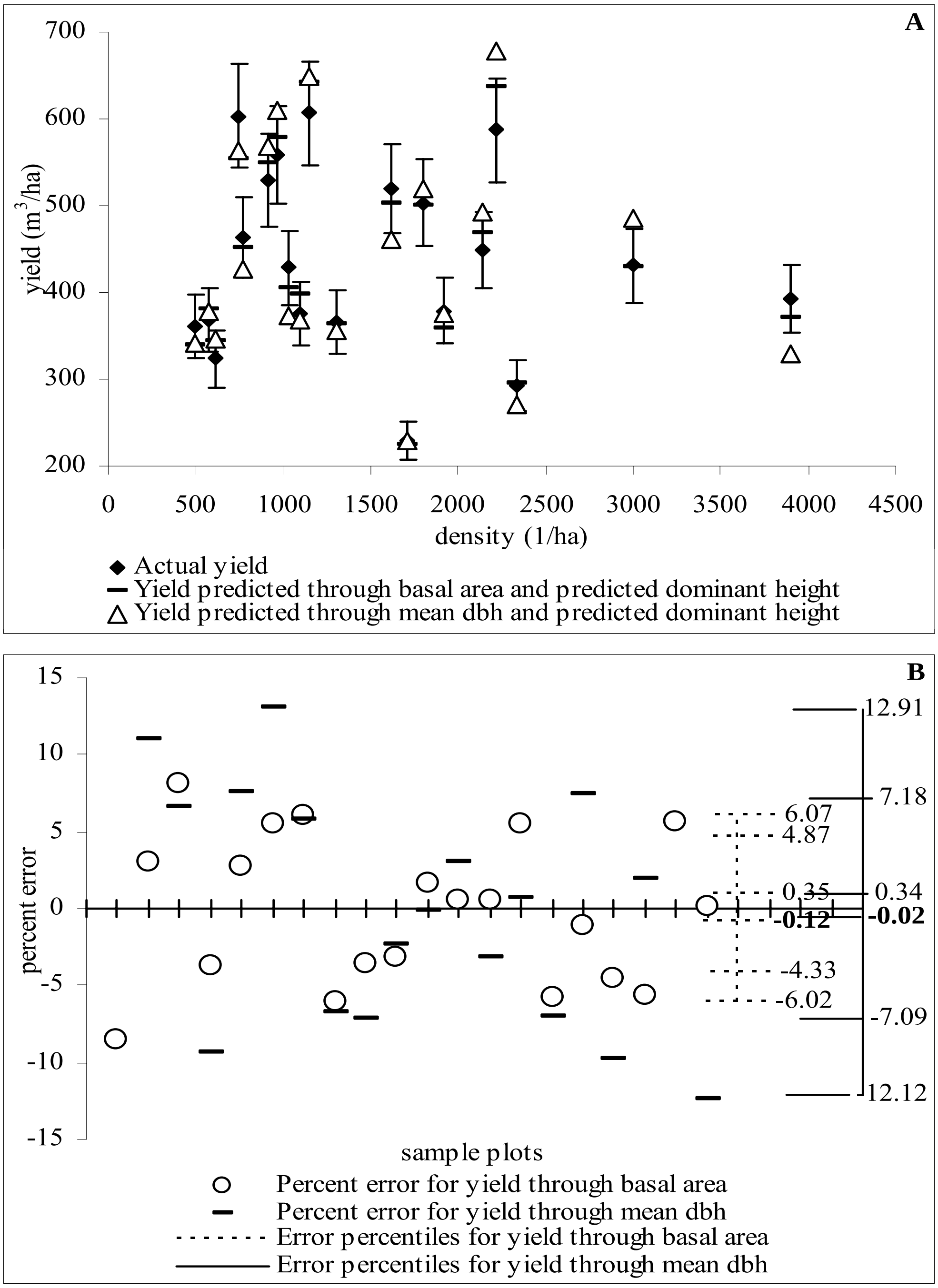


Figure 7 - Estimation of yield by the Equivalent diameter curves through **predicted** dominant height: **A**. Actual yield ±10% vs. predicted yield by the Equivalent diameter curves; **B**. Error percentages for predicted yield by sample plots (the values for 10th, 25th, 50th, 75th, 90th percentiles and the **mean** are indicated)

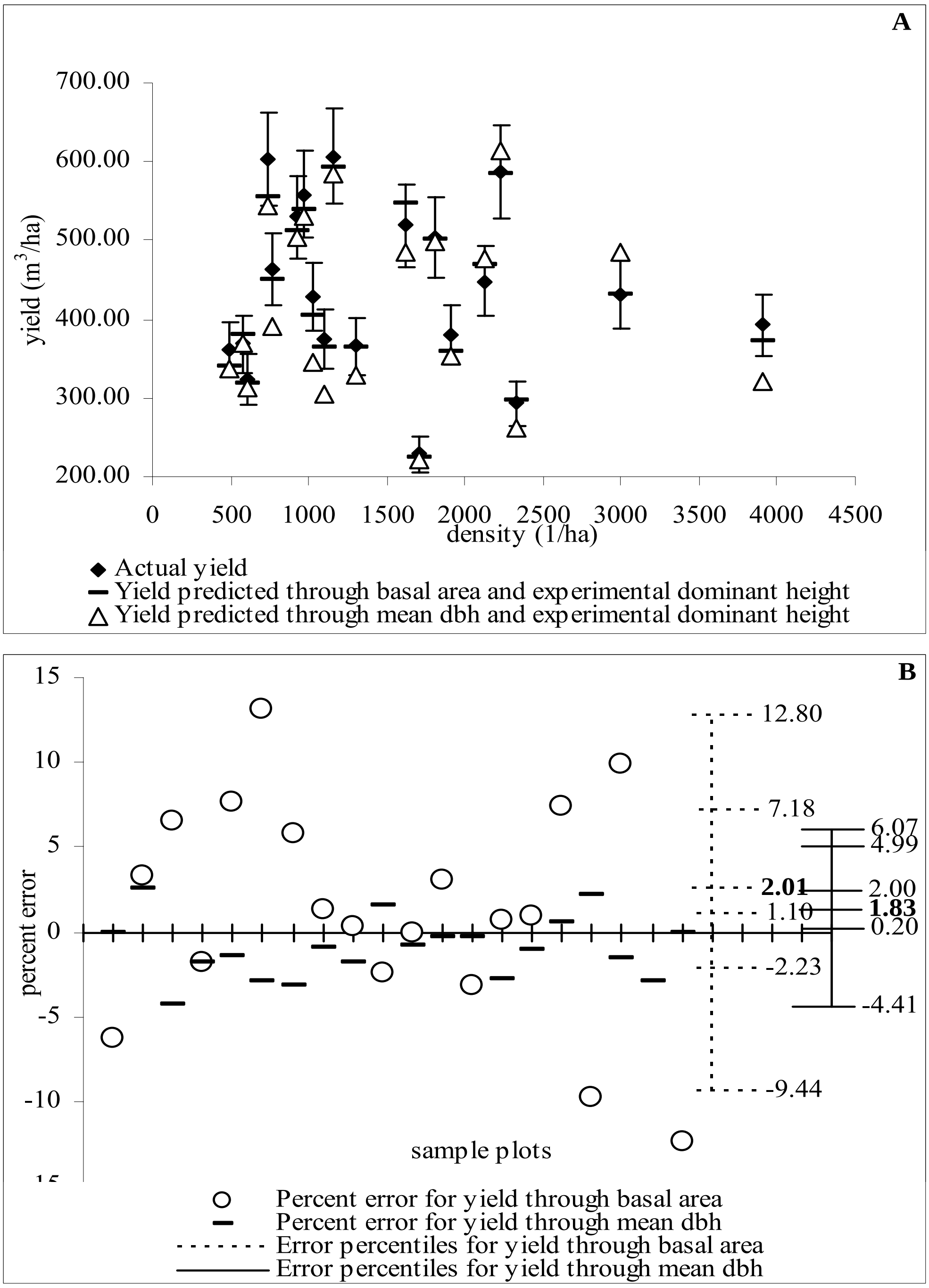


Figure 8 - Estimation of yield by the Equivalent diameter curves through **experimental** dominant height: **A**. Actual yield ±10% vs. predicted yield by the Equivalent diameter curves; **B**. Error percentages for predicted yield by sample plots (the values for 10th, 25th, 50th, 75th, 90th percentiles and the **mean** are indicated)

The validation data set allowed estimation of the characteristics density, mean breast height diameter and yield according to the Tables and their comparison with the experimental values (Eq. 17). The main descriptive statistics of the error estimates for both growth tables and the model of SDCD are shown in Table 2. For all examined growth parameters, the model of SDCD proved better goodness of fit, which is seen mainly from the narrower range and much smaller values of standard deviation and of standard error of mean of the estimated errors. The growth and yield tables exhibited general tendency to underestimation of yield and density. The estimated values of mean breast height diameter, on the other hand, revealed tendency to underestimation in the case of growth tables for the plantations and slight tendency to over estimation when the growth and yield tables for natural stands were employed (Table 2). The large error deviations in the case of parameter estimation through the growth tables can be explained by 2 main disadvantages of this type of models. The first disadvantage is that the stand density is averaged by growth stages within the site index class. Another weak point is the way of determination of the site indices, which are classified into several relative categories according to age and mean height, instead of the absolute site indices, which are determined as the dominant height at a given reference age.

Conclusions

The results from the present investigation showed that Equivalent mean diameter curves are excellent predictors for all studied growth parameters (density, breast height diameter and yield), having smaller relative errors than the Equivalent height curves.

Table 2 - Descriptive statistics for the estimated error (%) by SDCD model compared to the Growth and yield tables

Predicted Variable	Minimum	Maximum	Mean	Standard Error	Standard Deviation
Density according to the Tables by Tsakov (1983)	*-103.63*	*52.31*	*7.05*	*9.48*	*42.38*
Density according to the Tables by Nedyalkov (1983)	*-35.02*	*60.12*	*29.95*	*5.90*	*26.37*
Density by Eq. diam. curves (experim. H_{dom})	**-8.79**	**5.15**	**-1.99**	**0.82**	**3.68**
Density by Eq. diam. curves (pred. H_{dom})	-8.79	7.88	-0.10	1.09	4.87
Density by Eq. height curves (experim. H_{dom})	-80.75	53.55	7.73	7.75	34.67
Density by Eq. height curves (pred. H_{dom})	-80.75	37.24	-2.71	7.99	35.71
Mean dbh according to the Tables by Tsakov(1983)	*-25.10*	*34.88*	*8.68*	*3.69*	*16.50*
Mean dbh according to the Tables by Nedyalkov (1983)	*-31.77*	*17.52*	*-0.87*	*2.45*	*10.97*
Mean dbh by Eq. diam. curves (experim. H_{dom})	**-4.30**	**2.61**	**-0.97**	**0.41**	**1.82**
Mean dbh by Eq. diam. curves (pred. H_{dom})	-4.30	4.02	-0.02	0.54	2.43
Yield according to the Tables by Tsakov (1983)	*-51.84*	*50.23*	*20.65*	*5.26*	*23.51*
Yield according to the Tables by Nedyalkov (1983)	*7.26*	*56.64*	*35.47*	*2.89*	*12.92*
Yield by Eq. height curves (experim. H_{dom})	-51.14	17.15	-8.44	4.14	18.50
Yield by Eq. height curves (pred. H_{dom})	-28.93	17.15	-2.88	2.81	12.56
Yield by Eq. diam. curves through basal area (pred. H_{dom})	-8.55	8.08	-0.12	1.09	4.86
Yield by Eq. diam. curves through mean dbh (pred. H_{dom})	-15.68	16.11	-0.01	1.99	8.89
Yield by Eq. diam. curves through basal area (experim. H_{dom})	**-5.43**	**8.08**	**1.83**	**0.80**	**3.57**
Yield by Eq. diam. curves through mean dbh (experim. H_{dom})	-12.38	16.11	2.01	1.60	7.14

Among all six examined ways of determination of the stand stock as a growth parameter of highest interest, the Equivalent mean diameter curves based on the experimental values of basal area and dominant height showed best performance, which can be considered for the practical application of the SDCD. Another result of practical importance is the insignificant difference between the direct (experimental dominant height) and the indirect (predicted dominant height) way of parameter estimation. This finding implies that mean height, which is readily available in the forest inventory plans, along with the dominant height, can be successfully employed as an input variable for the SDCD application by using of the established mean-dominant height relationship. The proposed model of SDCD avoids some disadvantages of the existing models and supplies a reliable method for stand growth parameter evaluation.

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