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Principles of Deterministic Spatial Interpolators

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Resumo: A interpolação espacial é o processo de prever o valor de atributos em locais não amostrados a partir de medições realizadas em localizações diversas de uma determinada região [Burrough, McDonnell, 1998]. Rever e comparar os interpoladores determinísticos espaciais usados em Sistemas de Informação Geográficos (GIS) como a B-Spline, Fourier, TIN, IDW e as superfícies de tendência polinomiais é o objectivo principal deste artigo curto. Alguns aspectos técnicos computacionais são igualmente examinados.

Palavras-chave: Interpolação espacial, TIN, Superfícies de tendência global, Fourier, B-Spline.

Abstract: Spatial interpolation is the process of predicting the value of attributes at unsampling sites from measurements made at point locations within the same area or region [Burrough, McDonnell, 1998]. To review and to compare spatial deterministic interpolators used in Geographical Information Systems (GIS) such as B-Splines, Fourier, TIN, IDW and polynomial trend surfaces is the main goal of this short article. Some computational technical aspects are examined, as well.

Keywords: Spatial interpolation, TIN, Global polynomial trend, Fourier, B-Splines.

1. Preamble

For years, GIS studies have been expensive and unwieldy, with much of the analysis performed "by hand" using paper maps. Today, new technologies, such as

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GPS and remote sensing are helping the floodplain managers, for instance, to create accurate and current floodplain maps with improved efficiency and speed at a reasonable cost for avoiding severe social and economic losses from floods [Shamsi, 2008]. For instance, extensive GIS data were utilized to perform the hazard risk assessment by the City of Charlotte and Meclkelnburg County, USA. Remarkably, there were more than 2,500 structures in the 100-year floodplain. A GIS layer of the existing floodplain and floodway boundaries was, then, created to facilitate the flood hazard evaluation and insurance cost analysis.

Figure 1: Before and after Landsat imagery for the Mississippi flood of 1997 [Shamsi, 2008].

Still, spatial autocorrelation, interpolation, extrapolation and simulation are behind this study. For Mother Nature is quite common observation that, on average, values at points close together in space are more likely to be similar than points further away. This is known as Tobler First Law of Geography. It is the capability to use this local information, to provide a more complete description of the way an attribute varies within the area that really makes the difference among spatial interpolation methods.

To review in detail these different deterministic interpolation approaches is the main goal of this research. Thus, this short paper is divided into three major sections. Section 2 focuses on deterministic interpolators such as Triangulated Irregular Network (TIN), trend surface, Fourier, Inverse Distance Weight (IDW) and B-Spline. The last section reconsiders a short reflection of geography issues and spatial interpolation context.

2. Deterministic Interpolators

2.1. Triangulated Irregular Network

In this approach, the sample points are connected by lines to form triangles and, within each one, the surface is usually represented by a plane. It assumes variations only at borders. Yet, the removal of systematic differences before continuous interpolation and the quick assessment with sparse data are, thus, two intrinsic features of drawing boundaries. With Voronoi or Thiessen vector polygons, the best information about an unvisited point is the nearest data prediction point, a weighted linear combination approach where all weights are given to the closest sample. Therefore, polygon output depends on the sample layout, a point-in-polygon computation problem resolved by the semi-line algorithm (the pair line linkage of neighboring points is bisected at right angles through the polygon boundary). Hence, there are never any null regions while its size varies inversely with the point density.

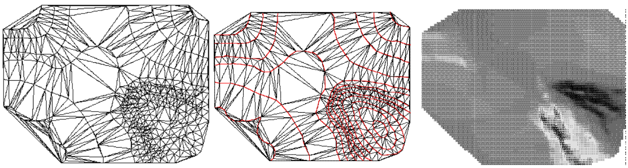

Figure 2: TIN model (left), contour lines (center) and surface shading (right) [Source: www.geog.buffalo.edu, 2008].

The ESRI Geostatistical Analyst® extension refers to this tool to analyze stationary and local outliers on the basis of the cluster assignment rule. After cells are placed into five classes, if the class interval is different from each of its neighbors then the cell will be colored gray to distinguish it from its neighbors. Griffith and Layne [1999] mention the use of Thiessen polygon surface to construct the geographical connectivity matrix. With their Canada boreal forest research, Nalder and Wein [1998] ranked, in terms of the mean absolute error, Voronoi third best for temperature and fourth for precipitation out of seven methods, lending support to the principle that complex methods are not inevitably more accurate.

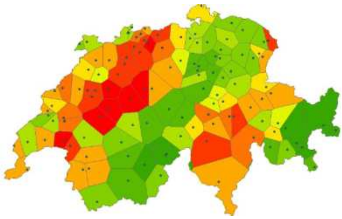

Figure 3: Thiessen polygon example
[Source: www.geo.unizh.ch/gis/teaching/courses/gis2/ws0708/unterlagen/pdf/V5_SpatialInterpolation.pdf, 2008].

According to Goovaerts [1999], TIN has been applied successfully to environmental modeling, categorical data (land cover like buildings, grass or wood) and elevation-erosive estimation (with its Algarve Portugal dataset, the erosivity linear equation equals 968.2+3.8087×elevation with ρ=0.75) although the assignment of local quantities can be a major difficulty. Nested tessellation is another possibility. Tobler's pycnophylactic interpolation also addressed this problem by creating a continuous interpolator that removes the changes of abrupt boundaries on the basis of the mass-preserving reallocation approach from primary data. This view assures that the attribute volume within the spatial entity remains the same. In conjunction with ancillary data, therefore, both methods can be useful for areal interpolation.

2.2. Polynomial Trend Surface

A polynomial trend surface, an inexact global basis model based on spatial coordinates, is used as a fitting regression procedure of a global surface for smoothing, filtering and data interpolation by separating the study variable into two components: large-scale variation (trend or regional features) and random error (non-systematic white noise fluctuation due to local features). Developed by Whitten in 1957, the unknown b_i coefficients of the coordinates polynomial regression are found by solving a set of simultaneous non-linear equations of cross products sums of the (x,y) coordinates and y_i values. If the *n* terms presented in **X** matrix correspond to the coordinates data points and the *n* terms of vector **Y** equal the correspondent values then b_0 of vector **B** represents the constant term while the remain are the regression coefficients of the polynomial (in terms of the following equation, the least-square solution equals $\mathbf{B}=(\mathbf{X'X})^{-1}(\mathbf{X'Y})$). Thus, the value of a particular (x_i,y_i) location equals $b_0+b_1x_i+b_2y_i$, if a linear polynomial is considered, or $b_0+b_1x_i+b_2y_i+b_3x^2_i+b_4x_iy_i+ b_5y^2_i$, if a quadratic one.

$$\overset{\mathbf{Y}}{\begin{vmatrix} y1 \\ \dots \\ yn \end{vmatrix}} = \overset{\mathbf{X}}{\begin{vmatrix} 1 & x1 & y1 \\ \dots & \dots & \dots \\ 1 & xn & yn \end{vmatrix}} * \overset{\mathbf{B}}{\begin{vmatrix} b0 \\ b1 \\ b2 \end{vmatrix}}$$

$$\overset{\mathbf{Y}}{\begin{vmatrix} y1 \\ \dots \\ yn \end{vmatrix}} = \overset{\mathbf{X}}{\begin{vmatrix} 1 & x1 & y1 & x1x1 & x1y1 & y1y1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & xn & yn & xnyn & xnyn & ynyn \end{vmatrix}} * \overset{\mathbf{B}}{\begin{vmatrix} b0 \\ b1 \\ b2 \\ b3 \\ b4 \\ b5 \end{vmatrix}}$$

Figure 4: The upper linear (a constant dip in a single direction) and lower quadratic (a bowl or dome shape) polynomial matrices.

It is this simplicity that makes this approach worth using instead of Kriging. “If with Kriging there is a price to pay with the estimation of variogram weights, the quadratic trend surface predictor uses up to six factors to describe the mean structure but it only needs one for the error structure” [Sarkozy, 1999]. “A high multicollinearity diagnosis is, however, a trend surface characteristic due to the strong functional relationship among the polynomial terms” [Anselin, 1992]. So, the indication of significant T-Student and R^2 indices should be suspicious. For instance, Anselin and O´Loughlin [1991] report the use of a quadratic trend surface regression to assess space importance as an explanatory variable for total conflict in Africa. In spite of the random white noise, a high multicolliniarity index and a significant T test were found among the residuals.

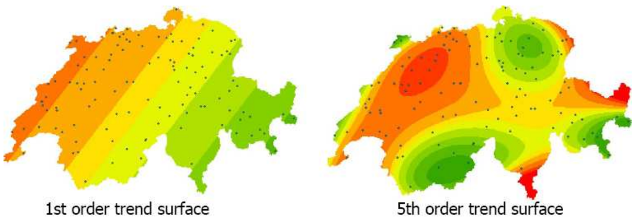

Figure 5: The first and fifth order polynomial trend surfaces
[Source: www.geo.unizh.ch/ gis/teaching/courses/gis2/ws0708/unterlagen/pdf/V5_SpatialInterpolation.pdf, 2008].

In terms of software, EcoSSe® develops this least square methodology quite well. With its organic matter dataset in the soil of Nebraska, USA, any spatial autocorrelation among the data was revealed. The Moran I correlogram was zero for all distances, the isotropic variogram was a pure nugget-effect and its cross-validation had a negative slope. Based on Anova variance, a cubic trend surface was then applied for this detrend operation while the variogram, Kriging and cross-validation estimation were estimated regarding the residuals. Certainly, this detrend operation illustrates the close link between trend surface analysis and Kriging.

Like any ordinary least square (OLS) approach, this method is highly susceptible to edge effects and rarely passes exactly through the original data points. Wang and Zhang [1999], for instance, report a tendency to accelerate without limit in areas like map edges where there is no point's control (an extrapolation issue). Another limitation regards uncertainty assessment. Finally, negative values can also be achieved depicting, thus, a serious distortion of reality.

2.3. Fourier

"The Fourier series is a mathematics-based technique for resolving any time domain function into a frequency spectrum much like a prism splitting light into a spectrum of colors" [Cross, 2000]. It is based on the discovery that it is possible to take any periodic time function and resolve it into an equivalent infinite summation of sine and cosine waves with frequencies that start at zero along multiples base frequencies. "In a clear way, the Fourier analysis transforms the data value using a series of sine's and cosines" [Isaaks and Srivastava, 1989].

"This global deterministic interpolator fits best with spatial datasets that exhibit marked periodicity such as ocean waves" [Walter, 2001]. Levine [1997] mentions this technique within the SASP® spatial pattern for Unix® platforms and exemplifies it with traffic flow shift over time for major city arterials. Cross [2000] reports this methodology for bird and insect type identification via seismographic information. Kratky [1981] describes its use in photogrammetry (a very cost-effective means of data vector capture) while Billingsley [1983] uses it with image raster analysis to enhance both short and long range pattern components. Curiously, Yao [1999] describes a new automatic cross-covariance matrix methodology between primary and secondary variables for CK estimation using Fast Fourier Transform and Bochner theorem.

Figure 6: A Spline example
[Source: www.geo.unizh.ch/gis/teaching/courses/gis2/ws0708/ unterlagen/pdf/V5_SpatialInterpolation.pdf, 2008].

"B-Spline local spatial inference model uses a radial basis polynomial to fit all available data point with minimum curvature in a continuous surface" [Walter, 2001]. Often used for smoothing digitized lines like soil maps, B-Splines predictions can also reach above the maximum or below the minimum available. Burrough and McDonnell [1998] approve this method for arc or chain representation within a vector model for saving data storage space at processing time expense. ESRI [2001] recommends it for digital elevation modeling (DEM) and labeling contour features while Miller [1997] reports its use with nitrogen concentration interpolation over space and time at Chesapeake Bay. "Yet, this adjustment may interpolate values whose nature has no relation with the original data but, rather, with the shape of how a polynomial adjusts to them" [Matos, 2001]. Concerning computational programming, B-Spline surfaces are defined in terms of blending functions that generate surface points as weighted averages of the point coordinates.

2.4. Inverse Distance Weighted

This exact deterministic method is a weighted average value at some known points. Like all distance weight averages, its common concern is the Krige regression effect (the conditional bias issue): under-estimation of high values and over-estimation of low values (smoothing out of the dips and the humps). The higher the function power, the more weight will be given to closer samples in conformity with Tobler's Law, where the best guess is the measured value at closest observations. Also, as more samples are included into the weighted linear combination, the resulting estimates become less variable, which leads to fewer extremes. Essentially, interpolation is smoothing.

Moving averages with a narrow window emphasize best spatial variation although, according to Goovaerts [1998], simulation maps reproduce best the spatial variability of the original data since a good estimation method will produce estimated values whose distribution is similar to the distribution of true values. For Zimmerman [1999], the recommended number of neighbors is around 6, while Declerq [1996] suggests the same for smooth surfaces but 20 for abruptly changing ones. Kravchenko and Bullock [1999] found no correlation between the number of neighbors to be considered and classical statistics parameters. "Usually, this is indicative of a stable situation, suitable for a weighted average estimation" [Clark and Harper, 2000]. Yet, with skew spatial datasets, the estimations were best accomplished with a power of 4 while low ones prefer a power of 1.

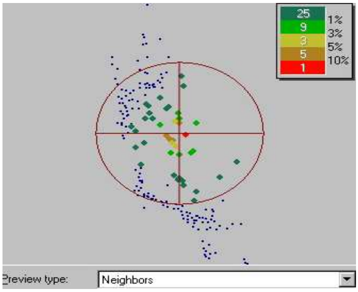

Figure 7: The search neighborhood of ArcGIS Geostatistical Analyst®. The points highlighted in the data view window give an indication of the weights that will be associated with each location in the prediction of unknown values.

The layout map also depends on data clustering (IDW works better if samples are evenly distributed) and outlier presence because this estimation is strongly influenced by the closest samples. Also, the reduction of data points should be mandatory in software for screened data. Anisotropy, an extra factor to consider, can also be resolved with the quadrant and the octant search.

Regarding programming, the resulting map first sets a grid over the area while the estimated value at each node is calculated. However, in some of those grid nodes, there are already samples implying an infinity value since distance equals zero. Therefore, a special case for this calculation should be made (for most software, the observed values are copied over forcing this technique to be an exact interpolator) as with the moon hole-effect: zero estimated value due to samples lacking within its neighborhood. Categorically, both factors may contribute to a bizarre choropleth map.

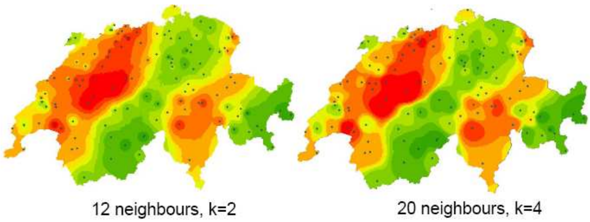

Figure 8: IDW estimation with $1/d^2$ (left) and $1/d^4$ (right)
[Source: www.geo.unizh.ch/gis/ teaching/courses/gis2/ws0708/unterlagen/pdf/V5_SpatialInterpolation.pdf, 2008].

Another dilemma is the uniform population distribution assumption within zones and how to choose the best weights for the weight function. A statistical search possibility for the optimal IDW is based on the cross-validation procedure. By using the original samples and their predictions, the best weight parameter is the one that holds the lowest root-mean-square prediction error. Hence, a more analytical approach is Kriging whose weights are optimized at each interpolation to produce a surface that satisfies minimum error variance. However, Dalthorp *et al.* [1999] present an estimation comparison study of the local mean for insect densities in golf courses using Kriging against IDW. On the basis of cross-validation errors, Kriging had a substantially higher MSE effectiveness than IDW on a particular golf course because of low sampling density. “Being complex does not imply better results” [Goovaerts, 2002].

3. Final Thought

Spatial data holds special features to the researcher (if performing statistical spatial analysis): Where does this occur? How does this pattern vary across the study area? How does an event at this location affect surrounding locations? Do areas with high rates of one variable also have high rates of another? Traditional statistical techniques tend to produce a summary statistic that quantifies the strength of a relationship within a dataset, for example. This approach is undesirable, from a GIS perspective, because it ignores the impact of space. It is important that spatial methods should explicitly incorporate the spatial component to develop a more sophisticated understanding from our data. Yet, four major disadvantages make spatial data special: Error propagation, spatial autocorrelation, the modifiable areal unit problem (MAUP) and ecological fallacy [http://hds.essex.ac.uk/g2gp/gis/sect72.asp, 2008]. Already beyond from the scope of this essay to explain these concepts, it is crucial that the reader understands that deterministic interpolation techniques are only a small issue regarding geographical science. Still, it is a critical part of it since spatial interpolation is an exclusive field of geographers.

Bibliography

Anselin, L. & O'Loughlin, J. (1992). Geography of International Conflict and Cooperation: Spatial Dependence and Regional Context in Africa. In Ficher (Ed.), *The New Geopolitics* (pp. 39-75). New York: Gordon and Breach Science Publication.

Anselin, L. (Ed.). (1992). *SpaceStat Tutorial. A Workbook for Using SpaceStat in the Analysis of Spatial Data*. Morgantown: Regional Research Institute, West Virginia University.

Billingsley, F.C. (1983). Data Processing and Reprocessing. *Manual of Remote Sensing*, Volume 1, 719-722.

Burrough, P. & McDonnell, R. (Ed.). (1998). *Principles of Geographical Information Systems*. Oxford University Press.

Clark, I. & Harper, W. (Ed.). (2000). *Practical Geostatistics*. Ecosse North America.

ESRI (Ed.). (2001). *Using ArcGIS Geostatistical Analyst*. ESRI Press.

Goovaerts, P. (1999). Using Elevation to Aid the Geostatistical Mapping. *Catena Research*, 34 (3), 227-242. Retrieved April 9, 2008, from http://www.sciencedirect.com/science.

Goovaerts, P. (2002). *Geostatistical Analysis of Environmental Data*. Lisboa, Portugal: ISEGI-UNL.

Griffith, D. & Layne, L. (Ed.). (1999). *A Casebook for Spatial Statistical Data Analysis: A Compilation of Analyses of Different Thematic Data Sets*. Oxford University Press.

Isaaks, E. & Srivastava, R. (Ed.). (1989). *An Introduction to Applied Geostatistics*. Oxford University Press.

Kravchenko, A. & Bullock, D. (1999). A Comparative Study of Interpolation Methods for Mapping Soil Properties. *Agronomy Journal*, 91, 393-400.

Levine, N. (1997). Spatial Statistics and GIS Software Tools to Quantify Spatial Patterns. *APA Journal*, 34, 456-471.

Matos, J. (Ed.). (2001). *Fundamentos de Informação Geográfica*. FCA-Lidel.

Nalder, I. & Wein, R. (1998). Spatial Interpolation of Climatic Normals: Test of a New Method in the Canadian Boreal Forest. *Agriculture and Forest Meteorology*, 92, 211-225.

Sarkozy, F. (1999). GIS Function Interpolation. *Periodical Civil Engineer*, 23, 63-86.

Shamsi, S. (2008). *GIS Applications in Floodplain Management*. Retrieved April 9, 2008, from http://gis.esri.com/library/userconf/proc02/pap0490/p0490.htm.

Walter, N. (Ed.). (2001). *Spatial Interpolation*. University of Calgary Press.

Wang, X. & Zhang, Z. (1999). A Comparison of Conditional Simulation, Kriging and Trend Surface Analysis for Soil Heavy Metal Pollution Pattern Analysis. *Journal of Environmental Sciences and Health*, 34, 73-89.

Yao, T. (1999). Nonparametric Cross-covariance Modeling as Exemplified by Soil Heavy Metal Concentrations from The Swiss Jura. *Geoderma*, 88, 13-38.

Zimmerman, D., Pavlik, C., Ruggles, A. & Armstrong, M. (1999). An Experimental Comparison of Ordinary and UK and Inverse Distance Weighting. *Mathematical Geology*, 31, 370-395.