

# EFFECT OF RELATIVE DISPLACEMENT AND NORMAL CONTACT LOAD ON FRETTING FATIGUE BEHAVIOUR OF Ti6Al4V ALLOY

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**ABSTRACT:** The main objective of this work is to study the fretting fatigue behaviour of Ti6Al4V in contact with a dissimilar mating material 34CrNiMo6 steel, using a sphere-on-plane configuration. There are a significant number of parameters that may affect fretting fatigue. Relative displacement amplitude and normal contact load are usually considered the most relevant ones. Thus, the role of relative displacement amplitude and normal load on fretting fatigue, life is presented. When the Ti6Al4V alloy is subjected to fretting fatigue the fatigue life drastically reduces as compared to the traditional plain fatigue case. In order to study this detrimental effect of fatigue life tests of both plain fatigue and fretting fatigue were carried out, highlighting the synergic effect between fatigue and fretting. Three different normal loads were used in the experiments. The relative displacement amplitude was changed by varying the tangential load. The fretting fatigue experiments were carried out in a fretting fatigue apparatus assembled on a servo hydraulic fatigue testing machine. In addition to the experimental work a nonlinear analysis using ABAQUS was carried out to determine the influence of the normal load on fretting fatigue behaviour. It was verified the strong dependence of fretting fatigue life on the two studied variables, namely the relative displacement amplitude and the normal contact load.

Keywords: fatigue, fretting fatigue, relative displacement amplitude, normal contact load.

**RESUMO:** O principal objectivo deste trabalho é o estudo do comportamento da fadiga com 'fretting' da liga de Ti6Al4V em contacto com uma liga de aço 34CrNiMo6, utilizando a configuração esfera - plano. Há um número significativo de parâmetros que podem afectar a vida de fadiga com 'fretting'. A amplitude de deslocamento relativo e a carga normal do contacto são considerados parâmetros mais relevantes.

Quando a liga de Ti6Al4V é submetida a fadiga com 'fretting' a vida da fadiga reduz drasticamente em comparação com o caso de fadiga tradicional. O objectivo deste estudo foi observar o efeito negativo do 'fretting' na vida de fadiga e para isso foram realizados testes tradicionais de fadiga e testes de fadiga com 'fretting'. Há que realçar os efeitos sinérgicos entre a fadiga e o 'fretting'.

Além do trabalho experimental foi realizada uma análise não linear com Abaqus para determinar a influencia da carga normal no comportamento da fadiga com 'fretting'.

Foi verificada a forte dependência da vida de fadiga com 'fretting' com as duas variáveis estudadas, nomeadamente a amplitude de deslocamento relativo e a carga normal de contacto.

**Palavras chave:** fadiga, fadiga com 'fretting', amplitude de deslocamento relativo, carga normal do contacto.

## 1. INTRODUCTION

Fretting is a complex phenomena, that involves many aspects of tribology, contact mechanics and material science. This phenomenon can occur whenever two components are in contact and between them it is a small relative displacement amplitude.

The term fretting can be classified in three categories: fretting fatigue (stick-slip regime), fretting wear (gross slip regime) and fretting corrosion [1]. Fretting is frequently accompanied by corrosion, so usually more than one type of failure can co-exist in the same contact [2]. Fretting fatigue is associated with the stick-slip (partial slip) contact

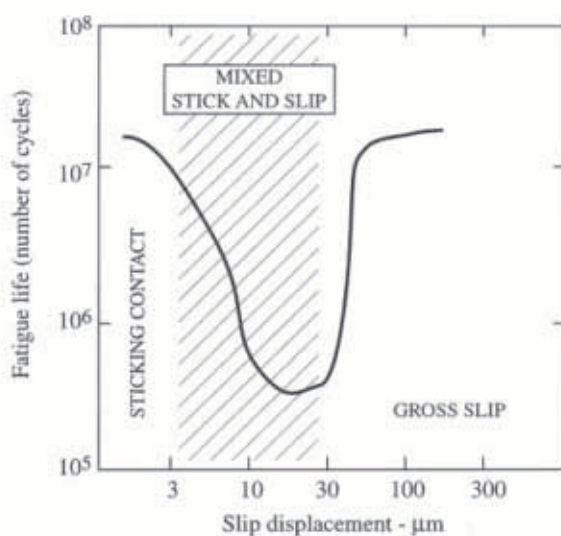
conditions and it is characterized by early crack nucleation and growth leading to failure. Fretting wear is associated with the gross slip contact conditions and it is characterized by permanent material loss. Fretting corrosion is a combined wear and corrosion process. Both type of failure are detrimental but fretting fatigue seems to be the most dangerous one. It results in a substantial reduction of fatigue life relative to plain fatigue.

The possibility to encounter this phenomena in mechanical components it is very high, for example medical implants, car components, airplane components that are subjected to vibration on the components connections [3]. Much attention has been given to fretting fatigue of aerospace materials in recent years because of the impact that

unfortunate catastrophic air disasters have on individuals and families.

Fretting fatigue behaviour of any material depends on various parameters/factors. Some researchers stated that almost fifty parameters may influence fretting damage [4,5,6], including: normal load, tangential load, axial load amplitude, material type, contact geometry, hardness, temperature, relative displacement amplitude, coefficient of friction, frequency [7].

Relative displacement amplitude has been recognized like one of the most important parameters in fretting fatigue behaviour which control crack initiation and propagation processes [8,9,10]. Jin and Mall named relative displacement amplitude in their study as a crucial parameter in crack nucleation and fretting fatigue life [7]. The mechanical surface damage induced by the relative displacement between the contacting bodies (the early initiation of the crack induced by tensile and shear stresses within the contact) reduces the fretting fatigue life compared with the plain fatigue life. Some of the researchers said that the ratio of the fretting fatigue limit to plain fatigue limit is between 20% and 30% [2,11,12,13]. The effect of the relative displacement amplitude in fretting fatigue life has been reported by Vingsbo and Soderberg [8]. They showed that fretting fatigue life initially is decreasing with the increasing of relative displacement amplitude, and afterwards a minimum fretting life is obtained at a certain point. It can be seen in figure that by increasing relative displacement amplitude range after this critical point the fretting fatigue life is increasing [8,14]. Relative displacement amplitude is controlled by many techniques: by changing the loads (normal contact load, tangential load or axial load), pad or specimen geometry. Relative displacement is often measured during the tests by extensometers and/or strain gages. The objective is to measure relative displacement between two points, one on the pad and the other one on the specimen [15].



**Fig. 1.** Fretting fatigue life vs. relative displacement amplitudes [8,14]

Normal contact load is another parameter that has effect on fretting fatigue life. In general if normal contact load is

increased the fretting fatigue life will decrease because of the monotonous increase in frictional stress amplitude with contact pressure. The reports in the technical literature regarding normal load and its influence on fretting fatigue life lead to contradictory conclusions, that normal contact load can have or not influence on fretting fatigue life.

Proudhon [16] showed that the crack nucleation condition was found to be mainly independent of the normal load. Ramalho [17] showed that the normal load does not have much influence on life. They observed only a small variation for a high contact pressure. Nakazawa [18] reported that fretting fatigue life decreased with an increase in normal load in an alpha brass, whereas a minimum life was observed at a certain normal load in aluminium alloy.

The objective of the present study was to investigate the effects of relative displacement amplitude and normal contact load on fretting fatigue life. To achieve this goal several fretting fatigue tests were carried out. Besides these, also some plain fatigue tests were carried out in order to highlight the synergetic effect between fatigue and fretting. Three different normal loads were used in the experiments and relative displacement amplitude was changed by varying the two loads that are measured during the tests. Fretting fatigue experiments were carried out in a fretting fatigue apparatus assembled in a servo hydraulic fatigue testing machine. In addition to the experimental work some nonlinear analyses using ABAQUS were carried out to determine von Mises stresses at the contact region in order to see the influence of the normal load on fretting fatigue behaviour.

## 2. EXPERIMENTAL PROCEDURE

### 2.1 Materials

It is well known that in the last years much attention has been given to the materials used in the aerospace industry. Thus the test specimen material is a titanium alloy Ti6Al4V and the pad material is a classical steel 34CrNiMo6. The Ti6Al4V alloy was also used for the plain fatigue test specimens. The chemical composition for Ti6Al4V alloy is given in Table 1 and for 34CrNiMo6 steel in Table 2. The mechanical properties of both materials are shown in Table 3. The approximate grain sizes of both materials are given in Table 4. The microstructures of these alloys are shown in the figure 2, which was obtained by Scanning Electron Microscopy (SEM) in the Laboratory of Electronic Microscopy of the Institute of Materials of the University of Minho (IMAT). Equipment: Leica Cambridge S360.

**Table 1.** Chemical composition (weight %) of Ti6Al4V alloy.

Elements	Al	V	Fe	Sn	Ni
Ti6Al4V	6.1	4.21	0,2	0,003	0,01

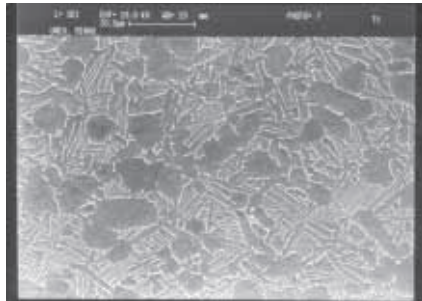
**Table 2.** Chemical composition (weight %) of 34CrNiMo6 steel.

Elements	C	Mn	Cr	Mo	Ni
34CrNiMo6	0,34	0,56	1,72	0,23	1,68

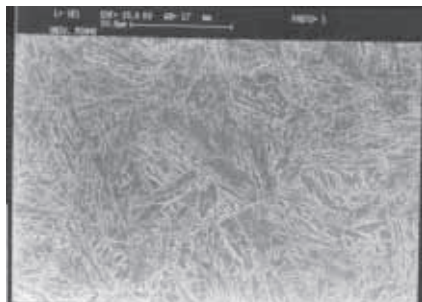
**Table 3. Mechanical properties of Ti6Al4V alloy and 34CrNiMo6 steel.**

Material	Mechanical properties			
	E (GPa)	$\sigma_{0.2}$ (MPa)	$\sigma_r$ (MPa)	$\epsilon_r$ (%)
Ti6Al4V	115	989	1055	16,1
34CrNiMo6	235	1101	1204	21.6

E – Young modulus; b –  $\sigma_{0.2\%}$  Yield strength;  $\sigma_r$  - Tensile strength;  $\epsilon_r$  - Rupture strain.



a



b

**Fig. 2.** The microstructures of a) Ti6Al4V; b) 34CrNiMo6

**Table 4.** The approximate sizes of grain of different materials

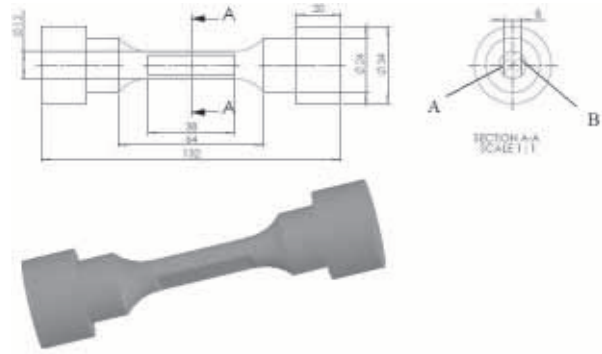
Ti6Al4V	34CrNiMo6
12 $\mu\text{m}$	15,6 $\mu\text{m}$

**2.2. Geometry of the specimen**

The fretting fatigue and plain fatigue experiments were carried out using the specimen shown in figure 3. It has a special shape with a round cross section and two flat sides (A and B). In this situation two zones of the contact were generated on the flat sides of the specimen. The cross section area of the specimen is 88,32 mm.

**2.3. Experimental test setup**

Figure 4 shows the fretting fatigue experimental test setup [19] used in this study along with the servo hydraulic uniaxial testing machine. The capacity of the servo hydraulic testing machine is 600 kN under static conditions and 500 kN for fatigue loading. The maximum cyclic loading frequency of the machine is 50 Hz. The equipment works at ambient temperature, in laboratory environment, but can be also used at low and high temperatures and at other environments.



**Fig. 3.** Geometry of the specimen used in fretting fatigue tests



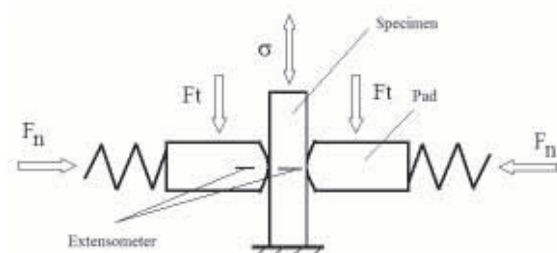
**Fig. 4.** Fretting fatigue experimental test setup [19]

**2.4. Experimental observations**

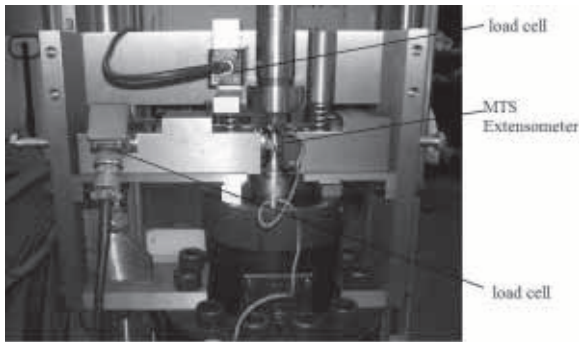
The fretting fatigue tests were carried out at a stress ratio,  $R=0,1$  in order to obtain the S-N curves. All tests were conducted at ambient temperature, in a laboratory environment, and at a cyclic frequency of 4 Hz. Also several plain fatigue tests were carried out to examine the effect that fretting have on fatigue life. These tests were also performed on the Dartec servo-hydraulic testing machine (figure 4) under a stress ratio,  $R = 0,1$ . Tests were carried out in laboratory air, 20° C, at a frequency of 8 Hz, considering, as fatigue strength, the complete specimens fracture.

Before fretting fatigue tests the machined specimens were polished with abrasive paper, finished with diamond spray (1  $\mu\text{m}$ ), and then cleaned in alcohol as to provide a standard surface.

Figure 5 shows schematically the test configuration in fretting fatigue tests. Figure 6 shows a detail of the fretting fatigue apparatus. A MTS extensometer was used to measure the relative displacement amplitude between the contact bodies (pads and specimen) as shown in figures 5 and 6.



**Fig. 5.** Schematic test configurations:  $F_n$ - pad normal load;  $F_t$ -pad tangential load;  $\sigma$  – machine axial load

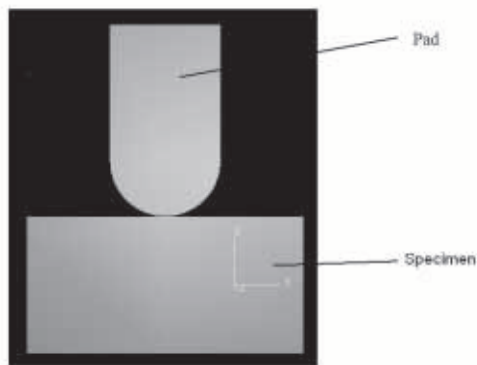


**Fig. 6.** Detail of the fretting fatigue apparatus

This extensometer has been adapted to the shape of the specimen and of the pads. One knife edge of the extensometer was attached to the pad where a cut has been made (so that the knife edge does not slip during the test), and the other knife edge was attached to the specimen. The extensometer was clamped to the specimen and to the pad with two metal springs.

### 2.5. Finite element analysis

In order to evaluate the influence of the normal load on fretting fatigue behaviour of Ti6Al4V, several finite element analyses (FEA) were conducted to determine the von Mises stresses at the contact region. The finite element model proposed is presented in figure 7. It is a bi-dimensional model that has only two components. The first body is the fretting fatigue specimen and the second body is the fretting fatigue pad.



**Fig. 7.** The two dimensional model: 1. specimen and 2. pad

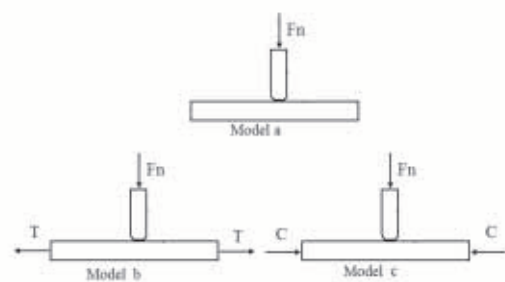
Some analyses were carried out only to see the influence of the normal load, after that several analyses were carried out when the specimen was subjected to an additional stress state either under compression or tension.

Schematically the finite element models proposed are presented in figure 8. The contact between specimen and pad was defined as a master-slave contact algorithm. Surface of the pad was defined as a slave contact surface and surface of the specimen was defined as master contact surface (Figure 9). The nodes on the slave surface cannot penetrate the segments that make up the other surface (the

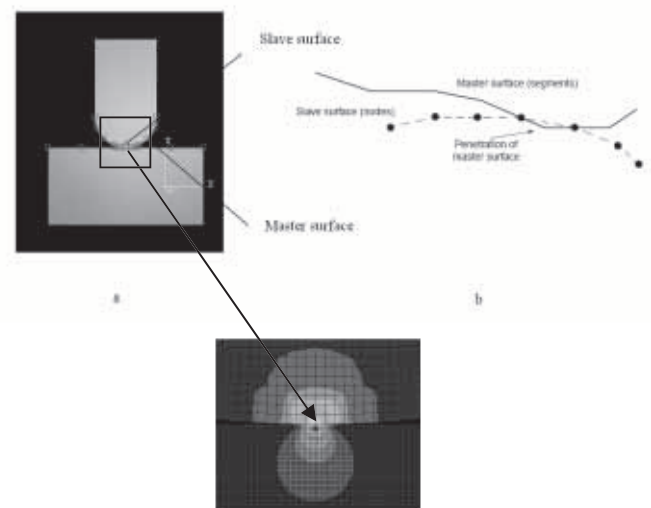
master). The algorithm places no restrictions on the master surface; it can penetrate the slave surface between slave nodes. A small sliding contact condition is used between the fretting specimen and fretting pad. This kind of contact condition is necessary for the type of analysis required for fretting fatigue, because this condition establishes the contact algorithm for the transfer of loads between the bodies in contact.

It is well known in finite element analyses that the correct choice of element for a particular simulation is vital if accurate results are to be obtained at a reasonable cost.

Thus for these analyses is adequate to used the 4-node bilinear, plane strain quadrilateral, reduced integration with hourglass control (CPE4R) Figure 10 shows a picture of CPE4R element [20].

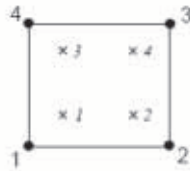


**Fig. 8.** Schematically the finite element models: Model a – normal load on the pad; Model b – normal load on the pad and additional tensile stress on the specimen; Model c – normal load on the pad and additional compressive stress on the specimen



**Fig. 9.** The master-slave contact: a. the present model; b. the master surface can penetrate the slave surface [20]; c. detail of the contact stress distribution.

The element size near the contact surface was  $8\mu\text{m} \times 8\mu\text{m}$ , and was gradually increased in the regions away from the contact zone. The finite element method, using the nonlinear formulation, was used to determine the Mises stress at the interface. The Young's modulus and Poisson's ratio used in the analysis, were 115 GPa and 0.345 for Ti6Al4V alloy and 235 GPa and 0.3, for 34CrNiMo6 steel.



**Fig. 10.** The 4-noded, plane strain elements [20]

The analyses that were performed on this model consisted of an initial step and two general analysis steps, as following:

1. In the initial step were applied the boundary conditions to regions of the model and was defined the contact between the two parts (specimen and pad) of the model.
2. In the first general analysis step the normal load was applied at the top of the pad to establish the contact between the fretting pad and specimen.
3. In the second general analysis step the additional load (tension or compression) was applied to the specimen.

The initial step is created by default by ABAQUS/CAE and the two general analysis steps were created.

Model a - is taking in consideration only the influence of the normal load on the pad and in this case were generated only the first two steps. For the other two models proposed were beside the normal load applied on the pad, an additional stress was applied on the specimen (compression or tension) all three steps were generated

**RESULTS**

**3.1. Fretting fatigue and plain fatigue tests**

The results are summarized in Table 5 along with the details of the conditions the fretting fatigue tests. It is reminded that several combinations of the normal load, tangential load and axial load were selected to provide a wide range of the testing conditions. The relative displacement value was changed by changing the tangential load. Table 6 shows the conditions and the number of cycles to failure for the plain fatigue tests.

Figure 11 shows the relation between the normalized maximum stresses used in fretting fatigue and plain fatigue tests, and the number of cycles to failure for Ti6Al4V alloy and Ck45 steel. The results for Ck45 steel were presented in other paper, see ref. [21]. The stress was normalized with respect to the tensile strength of material.

**Table 5** - Test conditions and results of the fretting fatigue tests

No.	La [kN]	Fn [N]	Ft [N]	d [μm]	Nf
Ti_1	7.0-70.0	1200	800	114.44	26466
Ti_2	6.2-62.0	1200	800	68.01	28491
Ti_3	5.5-55.0	1200	800	55.21	43330
Ti_4	5.5-55.0	950	800	37.26	50117
Ti_5	5.5-55.0	750	800	67.75	55322
Ti_6	4.8-48.0	1200	800	42.49	79862

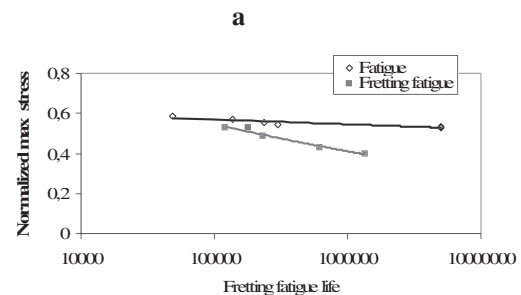
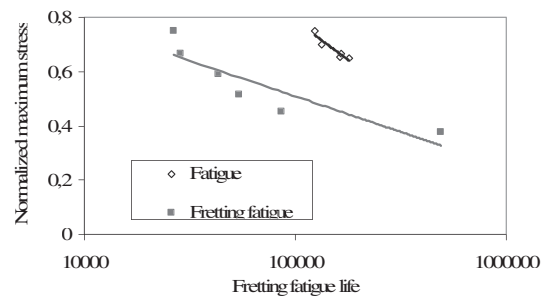
Ti_9	4.8-48.0	1200	600	29.38	71886
Ti_10	4.8-48.0	1200	450	24.18	103336
Ti_12	4.8-48.0	1200	800	100.26	54081
Ti_15	4.2-42.0	1200	800	83.66	86168
Ti_16	3.5-35.0	1200	800	40.32	490323

\* The tests were carried out without springs in vertical positions. The spring were replaced with two metal part allowing a bigger displacement between the two contacting bodies. La- axial load, Fn-normal load, Ft-tangential load, d- relative displacement amplitude, Nf- fretting fatigue life

**Table 6** - The plain fatigue tests conditions

Test	La [N]	s [MPa]	s <sub>r</sub> /s <sub>r</sub>	Nf
Ti_1	70000	792	0.751	123342
Ti_2	65000	735	0.697	133402
Ti_3	62000	701	0.665	165832
Ti_4	61000	690	0.654	162516
Ti_5	60500	685	0.649	179449
Ti_6	60000	679	0.643	2470301

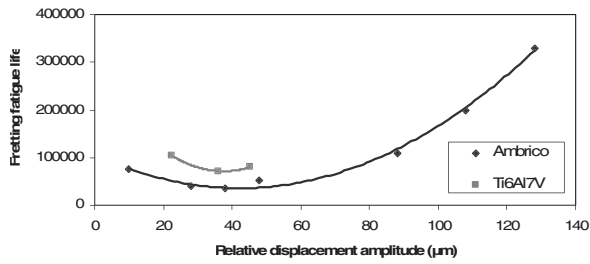
La - axial load, s-bulk stress, s<sub>r</sub>/s<sub>r</sub> – normalized max stress, Nf- number of cycles to failure



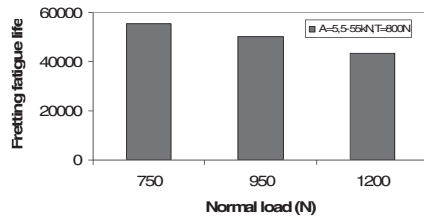
**Fig. 11.** The normalized maximum stress vs. fretting fatigue and plain fatigue life: a. Ti6Al4V alloy and b. Ck45 steel [21]

The effect of the relative displacement amplitude in fretting fatigue life is shown in figure 12. Here it is a comparison between the present results and the work carried out by Ambrico [14].

Figure 13 shows the variation of fretting fatigue life with normal load for one series of tests when the axial load is 5.5-55.0 kN and the tangential load was kept constant at 800 N (Tables 8).



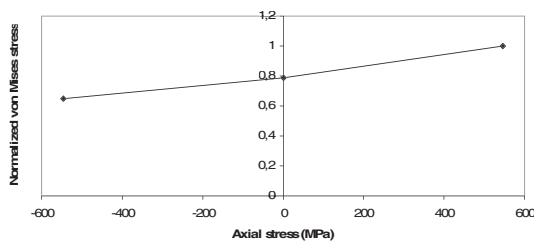
**Fig. 12.** Fretting fatigue life vs. relative displacement amplitude



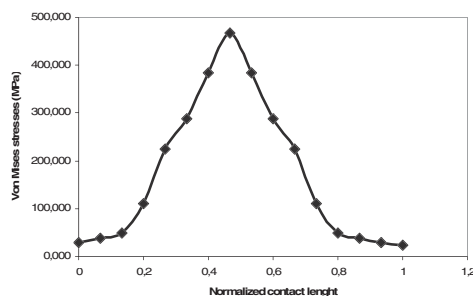
**Fig. 13.** Fretting fatigue live vs. normal load

### 3.2. FEA

Figure 9c shows an example of the contact stress distribution obtained by FEA. Figure 14 shows the plot of normalized von Mises stresses for all three models used in this study: Model a – normal load on the pin; Model b – normal load on the pin and additional tensile stress on the specimen; Model c- normal load on the pin and additional compressive stress on the specimen. Figure 15 shows how the von Mises stresses are varying with contact length.



**Fig. 14.** The normalized stress results from the numerical analysis



**Fig. 15.** The von Mises stresses vs. normalized contact length

## DISSCUSION

It is generally accepted that fatigue loading coupled with contact pressure between two components causes premature crack nucleation and eventually accelerates crack growth causing components to fail unexpectedly, even at stress levels below their plain fatigue limit or at fewer life cycles than predicted by plain fatigue analysis [22,24].

On Figure 11 it is clear the difference between the plain fatigue life and fretting fatigue life of the alloy used in this work. Lee and Mall [13] in their study concluded that the reduction of fatigue strength by fretting is generally attributed to the increase of tensile and shear stresses within the contact region and to surface or subsurface damage by the oscillatory relative movement. It is clear that is a bigger difference between fretting fatigue life and plain fatigue life on Ti6Al4V alloy and Ck 45 steel.

The difference between fretting fatigue life and plain fatigue life on Ti6Al4V can not be attributed only to the stress levels due to the contact stresses. The higher fretting effect should then be attributed mainly to the wear damage effect. T.C. Lindley [25] also reported that the damage evolution, namely through its wear performance, may be the responsible for the reduced fretting fatigue performance. An understanding of the wear damage evolution on fretting fatigue tests is therefore fundamental.

This part of the present study evidently shows that the magnitude of the relative displacement amplitude determines fretting fatigue behaviour under an applied axial load and normal load conditions. Vingsbo and Soderberg [8,22] found that the fretting fatigue life initially decreases with the increasing relative displacement, and a minimum fretting fatigue life is observed at a certain point. By increasing relative displacement range after this critical point the fretting fatigue life is increasing.

The same strong dependence was found in the present study, as shown on Figure 4. In this figure, after the work of Ambrico [14] it is also depicted the typical variation of fretting fatigue life with relative displacement for a wide range of variation of the latter parameter. The critical point it is about 40  $\mu\text{m}$ . There are studies that show for the same material different values for this critical point. For example, Jin and Mall in their study found that the critical relative displacement amplitude for Ti6Al4V alloy was between 50 and 60  $\mu\text{m}$  [7]. The relative displacement effect on fretting fatigue life is strongly material dependent and also depends on the test conditions (normal load, tangential load, pad geometry).

A number of studies have been reported the effect of normal load on fretting fatigue life. Some of the studies showed the normal load has influenced fretting fatigue life and some of these studies showed only a moderate influence of normal load. It is reasonable to suppose the effect of normal load on fretting fatigue life depends on the tests conditions (Ex: with/without springs, frequency etc). The present study shows that fretting fatigue life decreased by increasing the normal load. Some researchers state that this effect can be obtained only when the frequency is small (Ex: 1 Hz). If the frequency is 200 Hz, normal force has not effect in fretting fatigue life [23]. Nakazawa [18] reported that by increasing

the normal contact load the fatigue life is decreasing. A similar behaviour was obtained also by Ramakrishna. It has been showed that by increasing the normal contact load the fatigue life of Al–Mg–Si alloy AA6061 decreased [6]. In the present study for which the tests were carried out at a low frequency, namely 4 Hz showed that the normal load had an influence on the fretting fatigue life of the specimens.

It can be seen from FEA analysis (figure 14) that the local stress level (or contact pressure) is lower when the specimen is under compression. When the specimen is under tension the stress level is increasing.

## CONCLUSIONS

The Ti6Al4V alloy was tested under plain fatigue and fretting fatigue tests conditions.

From the present investigations the following conclusions can be drawn:

- i) it was depicted the typical variation of fretting fatigue life with relative displacement: it is decreasing in the beginning and then is increasing.
- ii) the damage induced by fretting increased the fatigue damage resulting in a shorter life of the component (specimen).
- iii) the Ti6Al4V alloy is very sensitive to fretting fatigue than Ck45 steel, this been justified by the well known poor tribological characteristics of Ti alloy.
- iv) at a low frequency used in this study (4Hz) the normal load has an influence on fretting fatigue life.

## ACKNOWLEDGEMENTS

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