ANALYTICAL AND NUMERICAL MODELLING OF PLASTICITY-INDUCED FATIGUE CRACK CLOSURE NEAR COLD-EXPANDED HOLES IN AIRCRAFT STRUCTURES

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ABSTRACT: Over the last few years considerable effort has been put into the improvement of the fatigue life of aircraft structures. Aircraft experience complex loading conditions during flight and safe operation requires understanding of the underlying mechanics of fatigue crack growth. One of the outcomes of the research carried out over the last thirty years is that fatigue cracks in metals are partially closed over part of a load cycle. This phenomenon is thought by many researchers to be the key to understanding the effect of non-uniform loading. Understanding crack closure is particularly challenging when initial residual stress fields (e.g. due to manufacturing or mechanical treatment) need to be taken into account. For example, rivet holes are a critical area for fatigue and they are usually cold expanded to create a beneficial residual stress field and to improve the fatigue performance.

This paper describes the development of a simple analytical model for plasticity-induced fatigue crack closure taking into account the residual stresses due to cold expansion of rivet holes. The model is compared to a more sophisticated finite element analysis of plasticity-induced crack closure. The results show that the residual stress field has a strong influence on the closure behaviour and therefore on fatigue crack propagation. The potential for the application of this model to real components is assessed by modelling some experiments taken from the literature. The model results agree with the experimental findings for the location of fatigue crack initiation. Residual stresses from FE analyses of cold expansion were used as an input to the analytical closure model, which successfully predicted crack propagation from cold-expanded holes. The results obtained show that this approach has potential for use as a life prediction technique in design.

Keywords: Cold-expanded holes, Residual stresses; Fatigue crack propagation, crack closure**.**

RESUMO: Ao longo dos últimos anos tem sido feito um esforço considerável para melhorar a resistência estrutural dos aviões a fenómenos de fadiga. Durante um vôo os aviões sofrem condições de carga complexas, nestas condições para que o seu funcionamento seja seguro é necessário que se percebam os mecanismos fundamentais que governam a propagação de fendas de fadiga. Um dos resultados da investigação efectuada ao longo dos últimos trinta anos indica que em metais as fendas estão parcialmente fechadas durante parte de um ciclo de carga. Muitos investigadores pensam que este fenómeno é a chave para se perceber o efeito de carregamentos não uniformes. O fenómeno de *crack closure* é particularmente interessante quando se tem em consideração a existência de tensões residuais (ex. devidas a processos de produção ou tratamentos mecânico). Como exemplo, os furos de rebites são zonas críticas propícias a fenómenos de fadiga e são usualmente expandidos para criar um campo de tensões residuais para melhorar a sua resistência à fadiga.

Este artigo descreve o desenvolvimento de um modelo analítico simples para quantificar o fenómeno de *crack closure* induzido por plasticidade tendo em consideração tensões residuais devidas à expansão de furos de rebites. O modelo é comparado com análises de elementos finitos mais sofisticadas. Os resultados mostram que o campo de tensões residuais tem uma influência sigificativa no fenémeno de *crack closure* e consequentemente na propagação de fendas de fadiga. A potencial aplicabilidade deste modelo em componentes reais é avaliado através da modelação de trabalhos experimentais publicados na literatura. Os resultados obtidos com o modelo analítico estão em concordância com as observações experimentais no que diz respeito às zonas de iniciação das fendas. Os campos de tensões residuais obtidos através de análises por elementos finitos foram usados como dados de entrada no modelo analítico de *crack closure,* que previu com sucesso a propagação de fendas de fadiga a partir de furos expandidos. Os resultados mostram que esta técnica tem potencial para ser usado em gabinetes de projecto.

Palavras chave: Furos expandidos a frio, Tensões residuais, Propagação de fendas de fadiga, *crack closure*.

1. INTRODUCTION

Since the pioneering work of Elber [1] it has been recognised that plasticity-induced crack closure plays an important role in the mechanics of fatigue crack propagation. This phenomenon is found to be a result of plastically deformed material, which is left along the crack faces as the crack grows. As a consequence, the crack faces contact each other and the resulting contact stresses reduce the effective stress intensity factor at the crack tip and therefore the rate of crack propagation. The understanding of fatigue crack propagation close to structural details such as rivet holes is a primary concern in airframe design. Fatigue performance of holes has been successfully improved using different techniques of cold-expansion. The procedure consists of expanding a hole to a larger diameter (usually an additional 3-5% of the diameter) using a mandrel. During expansion, this creates an annular region of plastically deformed material, which gives rise to a residual compressive stress field when the mandrel is removed. The resulting residual stress field has a beneficial effect on the fatigue performance since it delays nucleation and retards crack propagation [2-4]. As a crack propagates through the plastically deformed region around the hole, the amount of additional material left along the crack faces is expected to differ from that for a crack growing in virgin material. Some previous research has been carried out on the closure behaviour of fatigue cracks subject to residual stresses [5, 6], but the amount of numerical and analytical modelling devoted to cold expanded holes is limited. However, some recent work [7, 8] has been performed. LaRue and Daniewicz [6] characterized the closure behaviour of a cold-expanded hole using finite element analysis. Newman *et al.* [7] have also performed a similar study, and argue that plastic deformation induced by cold working is similar to that generated by an overload. In order to select the magnitude of the appropriate equivalent overload, a series of overloads were applied and that where the residual stress field is closest to that generated by cold-expansion was selected. Some recent experiments on fatigue crack propagation from cold-expanded holes have recently been reported by Pasta [8] for specimens with different levels of cold expansion tested at varying stress levels.

The aim of the present work is to undertake analytical and numerical modelling of plasticity induced fatigue crack closure close to cold-expanded holes. FE modelling of the closure behaviour is performed after initial modelling of the cold expansion procedure. This technique is accurate but time consuming and is not practical as a lifing technique. We have also, therefore, developed a more computationally efficient approach. This consists of an extension of Nowell's [9] plane stress model for plasticity-induced fatigue crack closure, which was initially formulated for a finite crack in an infinite plate. In the present paper it has been extended to an infinite plate with a circular hole and two symmetrical radial cracks. The effect of residual stresses is taken into account by including an extra term in the stress equations. A comprehensive study of the effect of the residual stresses on the closure behaviour is presented for constant amplitude loading at different levels of remote applied stress for a typical value of cold-expansion. The analytical model is also used to model recent experimental work reported by Pasta [8], which consists of measurements of fatigue crack growth from cold expanded holes with different levels of cold expansion.

2. ANALYTICAL MODEL

The analytical model used is shown in Fig. 1. It follows a strip-yield approach, based on the Dugdale concept [10]. The geometry consists of an infinite plate with a central hole of radius (r_h) and two radial symmetric cracks of length (a). Yield is possible ahead of the crack along a thin strip of material from $x = r_h + a$ to $x = d$.

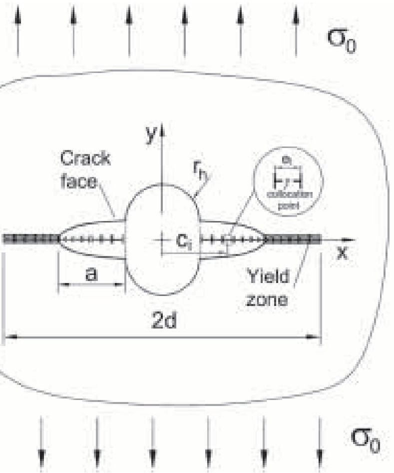


Fig. 1. Infinite plate with a central hole and two radial symmetric cracks under uniform remote applied stress σ_0.

The mathematical formulation is an extension of Nowell's [9] plane stress model for crack closure. This consists of an infinite plate with a finite central crack and is formulated using displacement discontinuity boundary elements. It has a similar physical basis as the models of Budiansky and Hutchinson [19], and Newman [14]. Solution is obtained by defining an objective function which is minimized, subjected to various constraints, using a standard quadratic programming approach. Three different sets of dislocation dipoles (i.e. displacement discontinuity boundary elements) are used. The first set represents the crack itself, the second represents the yield zone ahead of the crack along a thin strip (here from $x = r_h + a$ to $x = d$). The third set of dislocation dipoles is used only for modelling growing cracks, and represents the plastic wake which arises from the crack growing through the plastic zone. Material ahead of the crack is left with a residual stretch after a cycle of loading and this stretched material will remain as a plastic wake as the crack tip passes. In the current problem the argument of self-similarity cannot be employed, since there are two length scale parameters: the radius of the hole (r_h) and the crack length (a). Under these conditions the formulation proposed [9] for fatigue crack growing under general loading conditions must be used, even for constant amplitude loading, and the entire crack growth process has to be modelled.

2.1 Boundary conditions

The crack the faces may either be open and traction free, or closed and carrying a compressive stress, so that at a point j

$$\begin{array}{lll} \sigma_{yyj} = 0 & b_{yj} \geq 0 & \textit{crack open} \\ \sigma_{yyj} \leq 0 & b_{yj} = 0 & \textit{crack closed} \end{array} \qquad (1)$$

where b_y is the distance between crack faces. Similarly, in the strip where yield is permissible, three boundary conditions are possible during a particular load step: no yield, tensile yield or compressive yield. Tresca's yield criterion in plane stress is used, the boundary conditions in the yield strip are:

$$\begin{array}{lll} \left|\sigma_{yyj}\right| \le \sigma_{yield} & \Delta b_{yj} = 0 & \textit{no yield} \\ \sigma_{yyj} = \sigma_{yield} & \Delta b_{yj} > 0 & \textit{tensile yield} \\ \sigma_{yyj} = -\sigma_{yield} & \Delta b_{yj} < 0 & \textit{compressive yield} \end{array} \tag{2}$$

here, b_y is interpreted as the amount of stretch in the yielded strip, and Δb_y is the change in this quantity during a particular load step, since an incremental formulation of the problem is appropriate for the plastic behaviour.

2.2 Stress equations

The above boundary conditions are satisfied by distributing an array of displacement discontinuity dipoles, each of strength b_{yj}, along the crack and the yield zones (see Figure 1). The geometry studied in the present work requires the modification of the stress equation to use appropriate kernels for a dipole in an infinite plate with a hole. Taking advantage of the symmetry of the problem about $x = 0$, the stress σ_{yyj} produced at x_j by the boundary elements of equal strength, b_y, at c_i and $-c_i$ is,

$$\sigma_{yy}(x_j, r_h, c_i, e_i) = \frac{2\mu b_y}{\pi(\kappa+1)} \begin{pmatrix} K(x_j, r_h, c_i + \frac{e_i}{2}) - \\ K(x_j, r_h, c_i - \frac{e_i}{2}) + \\ K(x_j, r_h, -c_i + \frac{e_i}{2}) - \\ K(x_j, r_h, -c_i - \frac{e_i}{2}) \end{pmatrix} \tag{3}$$

where e_i is the length of each element (see Figure 1). The kernel functions, K in equation 3 are given in [11]. As in the original model of Nowell, n dislocation dipole elements are placed along the crack together with n more in the yield zone ahead of the crack. Elements near the crack tip are physically smaller to improve the accuracy of the results. Using Bueckner's superposition principle [12], the total yy component of stress is given by

$$\sigma_{yyj} = \sigma_{yyj}^{h} + \sigma_{yyj}^{res} + \frac{2\mu}{\pi(\kappa+1)} \begin{pmatrix} \sum_{i=1}^{2n} b_{yi} K(x_j, r_h, c_i, e_i) + \\ \sum_{i=1}^{n} b_{y0i} K(x_j, r_h, c_i, e_i) \end{pmatrix} \tag{4}$$

where σ_{yj}^{h} is the elastic stress distribution existing on the crack plane in absence of the crack and σ_{yj}^{res} is the correspondent residual stress component. The two last terms of equation (4) represent the effect of the dislocation dipoles along the crack and yield zone and in the crack wake respectively. For the geometry under analysis, an infinite plate with a central hole under remote tension σ_0, the stress component perpendicular to the crack plane is obtained from the standard solution [13].

$$\sigma_{yy}^{h} = \frac{\sigma_0}{2}\left(2 + \frac{r_h^2}{x^2} + \frac{3r_h^4}{x^4}\right) \tag{5}$$

The residual stress component, in cases where it is present, can have an arbitrary distribution and magnitude. Here we will focus on residual stress fields due to hole cold-expansion.

2.3 Objective functions and constraints used in the quadratic programming solution

The objective function used in the quadratic programming solution is defined in such a way that is always positive and is equal to zero when the boundary conditions are satisfied. As explained in detail in reference [9] the required objective function along the crack faces is as follows,

$$F_1 = \sum_{j=1}^{n} -(\Delta b_{yj}\sigma_{yyj}) - \left[(b_{yj} - b_{0yj})\Delta\sigma_{yyj}\right] - (\Delta b_{yj}\Delta\sigma_{yyj}) \tag{6}$$

Furthermore, the following constraints on stress and displacement are also used:

$$\begin{array}{ll} \sigma_{yyj} \le 0 & j = 1,\ldots,n \\ b_{yj} \ge b_{0yj} & j = 1,\ldots,n \end{array} \tag{7}$$

The objective function for the yield zone is

$$F_2 = \sum_{j=n+1}^{2n} \sigma_{yield}\left|\Delta b_{yj}\right| - \Delta b_{yj}(\sigma_{yyj} + \Delta\sigma_{yyj}) \tag{8}$$

An additional constraint on $\Delta\sigma_{yyj}$ in the yield zone is,

$$-\sigma_{yield} - \sigma_{yyj} \le \Delta\sigma_{yyj} \le \sigma_{yield} + \sigma_{yyj} \qquad j = n+1,\ldots,2n \tag{9}$$

The ability to model more complex load histories was incorporated by allowing the possibility of compressive yield of the stretched material on the crack faces. To do so, two sets of coincident boundary elements are defined along the line of the crack, as shown in equation (4). The first set, b_{yj}, models the crack opening, whereas the second set, b_{0yj}, models the stretched material in the wake and allows yielding in compression. This leads to the third objective function

$$F_3 = \sum_{j=1}^{n} -\left(\sigma_{yyj} + \Delta\sigma_{yyj} + \sigma_{yield}\right)\cdot\Delta b_{0yj} \tag{10}$$

The function to be minimized is the sum of the three objective functions, $F1 + F2 + F3$, given by equations (6), (8) and (10). To grow the crack, the length is increased by a small proportion and a complete load cycle is completed at each crack length in order to model the plastic zone correctly.

3. 2D FE MODELLING: RESIDUAL STRESS FIELD AND CLOSURE BEHAVIOUR

The aim of the finite modelling performed in this work is firstly to model the residual stress field due to cold expansion. Secondly, it seeks to model the closure behaviour of a crack growing from a hole with and without the influence of an initial residual stress field, allowing for comparison with the analytical model. Figure 2 shows the finite element mesh used.

3.1 Residual stress field in a cold expanded hole

The modelling of residual stresses resulting from cold expansion has been undertaken using analytical [14, 15] and numerical techniques [16, 17]. Here, the residual stress field was modelled using FEM, but reference is also made to results obtained using Nádai's analytical model [14]. Numerical modelling of cold expansion in 2D geometries is a relatively straightforward task consisting of two steps: 1) the hole is expanded by applying a given level of radial displacement (or internal pressure) to the edge of the hole 2) the nodal displacements are released and the residual stress field is created (see Figure 3). A similar procedure was used in reference [16] for modelling the residual stress field in a finite plate with central hole. The material properties used are representative of the titanium alloy Ti-6Al-4V: the yield stress is 1000 MPa, Young's Modulus 110 GPa; and Poisson's ratio 0.34. The results obtained using this technique are presented in Figure 4 alongside the analytical results obtained using Nádai's [14] model, reported in more detail in reference [11].

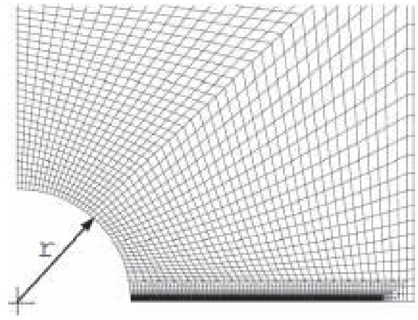


Fig. 2. Plate with a central hole with two radial symmetric cracks, mesh details:he plate; b) mesh detail close to the hole region (hole radius=2.5mm, half-width=100mm).

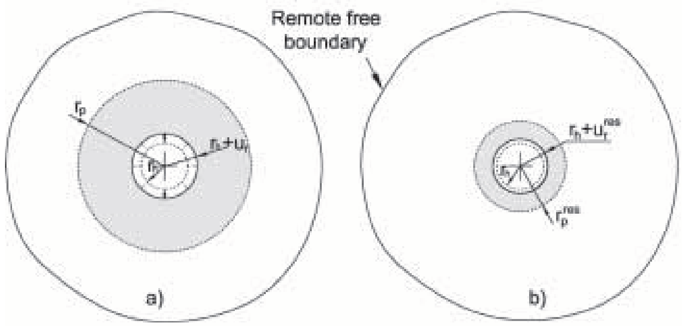


Fig. 3. Cold expansion process: a) after imposing hole edge displacement (or pressure); b) After removing the imposed hole edge displacement.

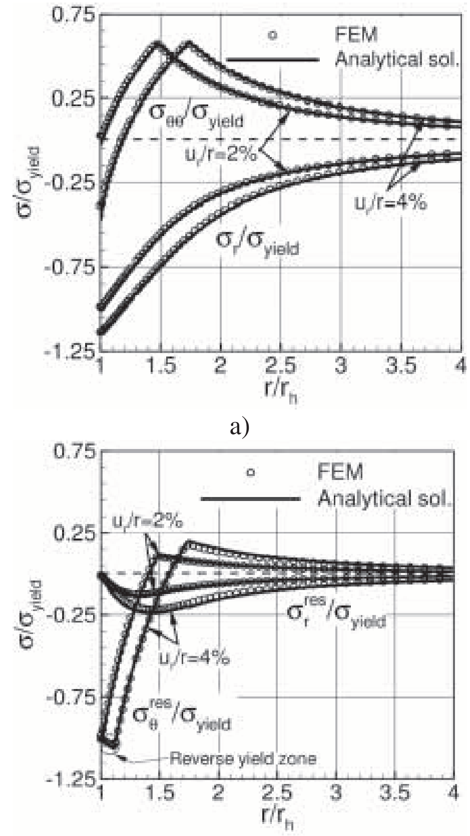


Fig. 4 Stress field induced by the cold expansion process: a) After imposing hole edge displacement; b) After removing the imposed hole edge displacement.

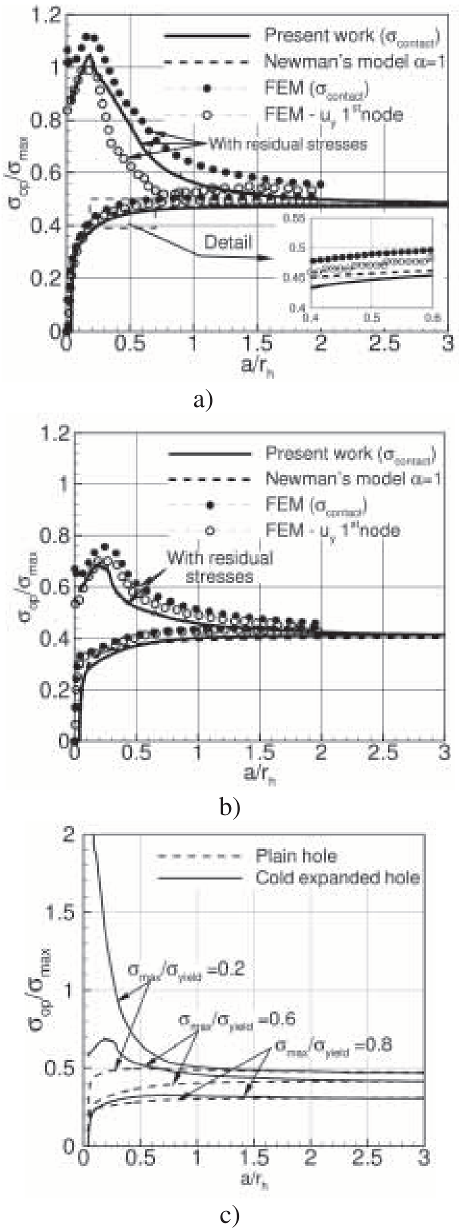
3.2 Opening stresses for holes with and without cold expansion

In order to simulate plasticity-induced fatigue crack closure using the finite element method, crack growth has to be modelled. In the present work crack growth was performed by node release, i.e. nodes along the crack plane are released sequentially by modifying the appropriate boundary conditions for the nodes. The crack is allowed to grow by one element every two load cycles. The possibility of contact between the two faces of the crack was taken into account by modelling a rigid line, and ascribing contact to the elements along the crack plane. The 'augmented Lagrange' contact algorithm available in ABAQUS [18] was employed. This uses a penalty function method for each iteration, allowing an interpenetration of 0.001% of the contact characteristic length. The material model used was elastic perfectly plastic with a von Mises yield criterion. The material properties used are the same as those presented in section 3.1. Two different techniques were used to calculate the opening load: 1) the node displacement method; 2) the contact stress method. The first method consists of monitoring the displacement of a node as the load is applied. The opening load is found when the displacement of the node monitored became positive during the loading stage of a load cycle and the closing stresses are found when the displacement of this node falls to zero during the unloading stage. The first node behind the crack tip was used for this purpose. In the second approach the contact stresses along the crack plane are used to calculate a residual stress intensity factor resulting from the compressive residual stresses which exist along the closed portion of the crack faces at minimum load. The total residual stress intensity factor, K_{res}, is found by summing the contributions (K_σ^i) from the individual (M) elements in contact [11],

$$K_{res} = \sum_{i=1}^{M} K_\sigma^i \tag{11}$$

where K_σ is given in reference [11]. The opening load is calculated by setting the net stress intensity factor due to both residual and applied loads equal to zero. This leads to the following equation,

$$\sigma_{op} = \frac{K_{res} + C \cdot \sigma_{\min} \cdot \sqrt{\pi \cdot (r_h + a)}}{C \cdot \sqrt{\pi \cdot (r_h + a)}} \tag{12}$$



a)

b)

c)

Fig. 5. Opening stresses for an infinite plate with a circular hole and two radial symmetric cracks propagating under constant amplitude loading (u_r/r=4% and R=0): a) $\sigma_0 / \sigma_{yield}$ = 0.4; b) $\sigma_0 / \sigma_{yield}$ = 0.6; c) $\sigma_0 / \sigma_{yield}$ = 0.2, 0.6 and 0.8.

4. APPLICATION OF THE ANALYTICAL MODEL TO A REALPROBLEM

As mentioned previously, a finite element approach such as that described above is very time consuming and it is therefore appropriate to develop a more efficient approach which may be applied in practical design studies. We therefore return to the analytical model described in section 2 with the intention of using it to predict fatigue crack growth from cold-worked holes. As well as comparing this approach with the finite element method, a specific objective will be to compare been crack propagation predictions from the model with experimental data recently reported by Pasta [8], who tested specimens with and without residual stress. The geometry of the specimens used in [8] and the cold expansion procedure are presented in Figures 6 a) and b) respectively. The process consists of pulling a mandrel through a pre-drilled hole. Following this, the hole is reamed. A split sleeve and a lubricant are used to protect the material at the hole edge from friction during the expansion procedure. Pasta performed been experiments for three expansion levels: 2.5%, 4% and 7% and at three different load levels (30, 32 and 35 kN) using a load ratio R = 0.1. The material used was the aluminium alloy 5083-H321, which has a yield stress of 255 MPa, an ultimate tensile stress of 360 MPa, a Poisson's ratio equal to 0.33 and Young's modulus of 70.2 GPa.

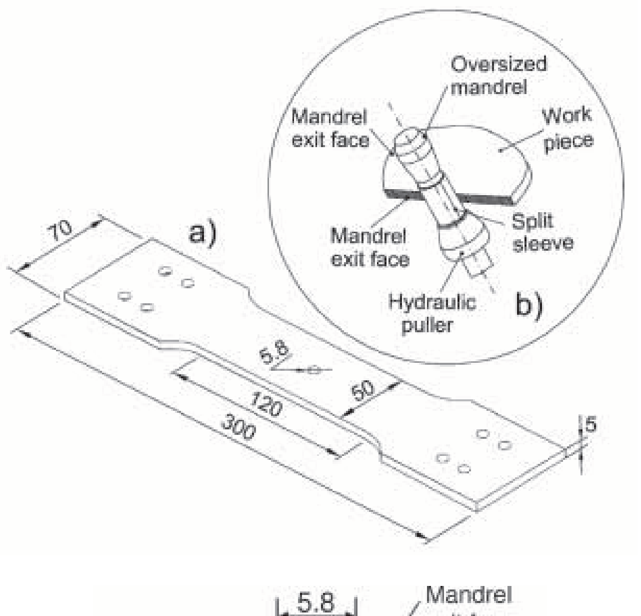


Fig. 6. a) Geometry of the specimen; b) Detail of the cold expansion process; c) Typical fracture surface of a cold expanded hole, after Pasta [8].

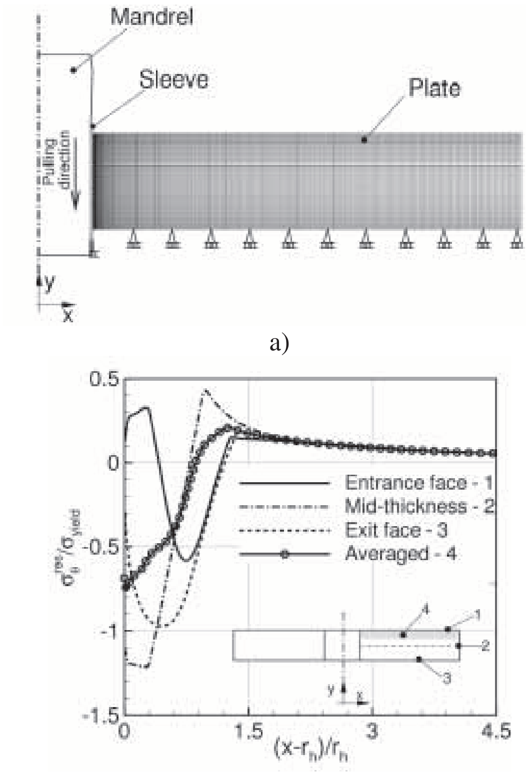
The analysis of the fracture surfaces of specimens with plain holes revealed that fatigue cracks propagated symmetrically and perpendicularly to the loading direction [8]. In the case of specimens with cold expanded holes the shape of the fatigue cracks was semi-circular as shown in Figure 6 c), with fatigue cracks initiating on the mandrel entrance face of the specimens.

4.1 Modelling of the residual stress field

Consideration of Pasta's results suggests that a model which accounts for the through thickness variation of residual stress is required, rather than the two-dimensional analysis presented in section 3.1. To account for through thickness effects a 2D axi-symmetric model was therefore developed. The advantage of using a 2D axi-symmetric model instead of a fully 3D model is that it is simpler and less computationally demanding, but still provide a good description of the through thickness residual stress field [19]. Figure 7 shows the finite element mesh used. The mandrel (assumed to be rigid), sleeve (steel) and plate were all modelled. The applied interference is given by:

$$i = \frac{D_m + 2t - D_i}{D_i} \times 100\,\% \qquad (13)$$

where D_m is the major mandrel diameter, t is the sleeve thickness and D_i is the starting hole diameter. The thickness of the sleeve was assumed to be 0.15 mm, a typical value according to a commercial catalogue. Since the residual stress profiles are to be used in the analytical crack closure model the material model was assumed to be the same (elastic perfectly plastic) and a frictionless contact between the mandrel and sleeve was also assumed. Figure 7 b shows the resulting circumferential residual stress for different sections through the thickness of the specimen.



a)

b)

Fig. 7. a) 2D finite element axi-symmetric model used to model the cold expansion process. b) Through-thickness residual stress field profiles. Circumferential residual stresses, *i* =4%.

4.2 Prediction of the closure levels using the analytical model

Figure 8 shows the opening stresses obtained using the analytical model presented in section 2. Four different applied loads are considered (30, 32 and 35 kN) using a residual stress profile which has averaged through the thickness. For comparison, additional results are presented for the case of no residual stress ('plain hole') and for 30kN using the residual stresses at the entrance face.

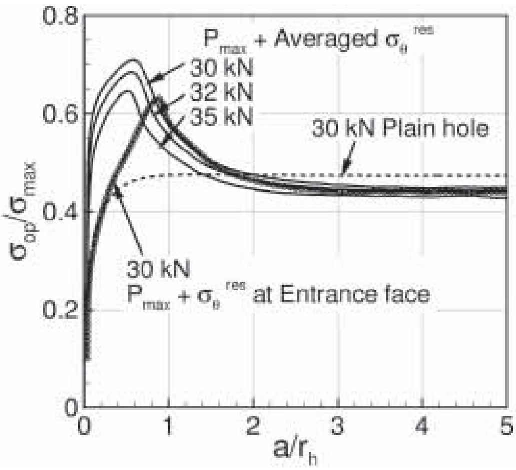
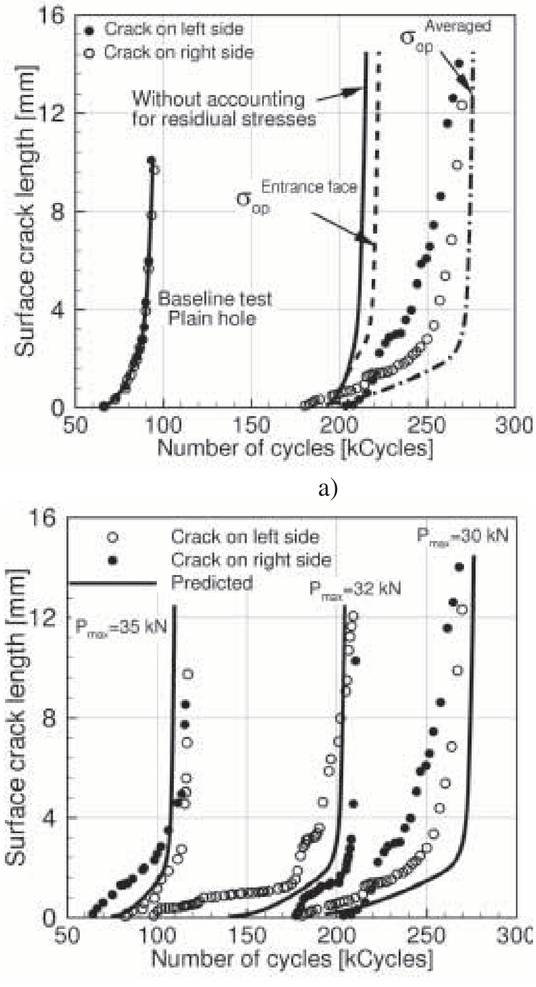


Fig. 8. Opening stresses for different levels of remote applied load, including the effect of residual stresses due to an hole expansion of *i*=4% and R=0.1.

4.3 Fatigue life prediction

In order to calculate the fatigue life of the cold expanded hole specimens the following assumptions were made: (1) The crack shape was assumed to be a semi-circular corner crack at the edge of a hole, and stress intensity factor solution used was the Newman and Raju [20] solution; (2) Fatigue cracks are assumed to be symmetrical, i.e. the same size on each side of the hole; (3) The material model was assumed to be elastic perfectly plastic; (4) The residual stresses due to cold expansion were calculated using a 2D axi-symmetric model; (5) The crack closure model is a 2D plane stress model for an infinite plate with two radial symmetric cracks at the edge of a circular hole; (6) The crack propagation law used is the Paris law [21] but modified to account for crack closure, $\frac{da}{dN} = C \cdot \left(\Delta K_{eff}\right)^m$.

The opening stresses profile, shown in Figure 8, for 30 kN plain hole does not take into account any initial residual stress field and is to be used to analyse the baseline test presented in Figure 9 a). The results of this analysis were used to find the Paris Law constants. C and m were found to be 1.016E-06 and 3.0284 (with da/dN in mm/cycle and ΔK_{eff} in $MPam^{0.5}$) respectively. As shown in Figure 8 the opening stresses depend on the initial residual stress field selected for input to the analytical model. Figure 9 shows experimental results obtained for a maximum applied load of 30 kN and R=0.1 for the case of a plain hole (baseline test) and a cold expanded hole *i*=4%. Predictions obtained using the Paris Law are also shown. The initial number of cycles used a starting point for these predictive curves was taken as the average of the number of cycles at which cracks were first measured on the left and right sides of the hole. In the case of the cold expanded hole, it can be seen that the prediction obtained without taking into account the initial residual stress field underestimates the fatigue life. Using the opening stresses from the entrance face, the fatigue life prediction improves slightly but is still poor. Finally, using the opening stresses obtained from the averaged residual stresses the prediction is good. Figure 9 b) compares the predictions based on the averaged residual stress profile for other loading conditions (32 and 35 kN). It can be seen that the analytical predictions are in good agreement with the experimental data.



a)

b)

Fig. 9. a) Baseline test and expanded hole i=4%, P_{max}=30kN and R=0.1; b) Comparison between experimental and predicted fatigue lives for 4% hole expansion and P_{max}=30, 32 and 35 kN with R=0.1.

5. CONCLUSIONS

An analytical model for plasticity-induced crack closure has been presented for the case of an infinite plate with a hole and two radial symmetric cracks. The existence of an initial residual stress field is also taken into account. The analytical crack closure model was compared to finite element simulations of plasticity-induced crack closure with and without residual stresses. In the absence of residual stresses, the analytical model compares well with Newman's [22] closure model. The residual stresses due to cold expansion were shown to have an important effect on the level of closure. As expected, this influence was shown to be stronger for lower levels of applied stress. In the last section of this paper the analytical model proposed in this paper was used predict fatigue crack propagation in specimens with cold expanded holes. The results obtained were in good agreement with experimental data recently published by Pasta [8]. The accurate modelling of the through thickness residual stress was shown to have a significant influence on the opening stresses, and therefore playing an important role in fatigue live prediction. The results suggest that two dimensional analysis should be used with care since the results obtained will depend on the residual stress profile. The values obtained from a 2D analysis may not be representative of the actual stress field in the crack initiation region.

6. ACKNOWLEDGEMENT

P. F. P. de Matos would like to acknowledge the support of the Portuguese *Fundação para a Ciência e a Tecnologia* (FCT) through a D.Phil. scholarship (reference SFRH/BD/1289/2003) financed by POSI.

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