An inventory model for deteriorating items with exponentially increasing demand and shortages under inflation and time discounting

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Abstract

A lot-size inventory model for deteriorating items is derived with exponentially increasing demand by allowing complete backlogging. The effects of inflation and time value of money are studied on the model. It is assumed that the units in inventory deteriorate over time at a constant rate. The inventory policy is discussed over a finite planning horizon with several reorder points. Sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out with the help of a numerical example.

Keywords: Deterioration, Inflation, Time-value of a money, Shortages

1 Introduction

Quite a good number of researchers are involved in developing inventory models for items deteriorating with time. Whitin(1957) derived a model of fashion goods deteriorating at end of the storage period. Ghare and Schrader(1963), Shah and Jaiswal(1977), Aggarwal(1978), Goel and Aggarwal(1981), Roy et al(1983) developed inventory models by assuming rate of deterioration of units in an inventory to be constant. Bahari-Kashani(1989), Covert and Philip(1973), Philip(1974), Mishra(1975), Deb and Chaudhuri(1986) derived inventory models with time dependent deteriorate rate. A survey of literature on inventory models for deteriorating items was given by Raafat(1991), Shah and Shah(2000).

Silver and Meal(1973) developed an inventory model for deterministic time-varying demand and gave approximate solution which is known as "Silver-Meal Heuristic". Donaldson(1977) solved the above model analytically. Ritchie(1984) discussed the exact solution for a linear increasing demand derived by Donaldson(1977).

Deb and Chaudhuri(1987) extended the model of Donaldson(1977) by allowing shortages. Goyal(1988), and Mudreshwar(1988) corrected Deb and Chaudhuri's(1987) calculations for shortage cost. Dave(1989) reconsidered the problem of Deb and Chaudhuri(1987) and proposed a method for solving it and reducing the necessary computational steps. An EOQ model was developed by Goswami and Chaudhuri(1991) considering a linear trend in demand, finite rate of replenishment and shortages. In (1991), they also discussed the above model for deteriorating items.

The above models were developed under the basic assumption that all the costs associated with the inventory system remain constant over time. These inventory models do not include inflation and the time-value of money as the parameters of the system. Buzacott(1975) was the first to develop EOQ model taking inflationary effects into account. A uniform inflation rate was assumed for all associated costs and expression for the EOQ was derived by minimizing the average annual cost. Almost at the same time Misra(1975) developed inventory model under aforesaid situation.

Bierman and Thomas(1977) discussed an inventory model considering both inflationary trends and time discounting. They assumed same inflation rate for all cost factors. A search procedure was suggested by the author to find the EOQ. A similar cost equation was developed by Misra and Wortham(1977). They suggested a method for finding an approximation to the EOQ. Later on Misra(1979) gave a model which incorporated the time value of money and different inflation rates for various costs associated with an inventory system. The optimal order quantities obtained with and without time discounting and inflation were quite different, eventhough, the corresponding total cost per time unit were almost the same. Jeya Chandra and Bahner(1985) extended Misra(1979)'s model by allowing shortages and a finite replenishment rate. In all these models, the deterioration of units in an inventory system was not considered.

In the present paper, we have developed a deterministic inventory model for items with exponentially increasing demand. Shortages are allowed. It is assumed that units in inventory deteriorate at a constant rate over time. The time-value of money and different inflation rates for various costs associated with the inventory system are considered. The sensitivity analysis is carried out with the help of a numerical example. The paper concludes with a conclusion section.

2　Assumptions and notations

The mathematical model is derived with the following notations and assumptions:

1. $R(t) = ae^{bt}$, a, $b > 0$, $a >> b$ is the demand rate at any time t.

2. A is internal ordering cost.

3. The replenishment rate is infinite.

4. The cost components are divided into two classes.

   The costs that increase at the inflation rate occurring within the company are brought under class I. Whereas the costs increasing at the inflation rate of the general economy
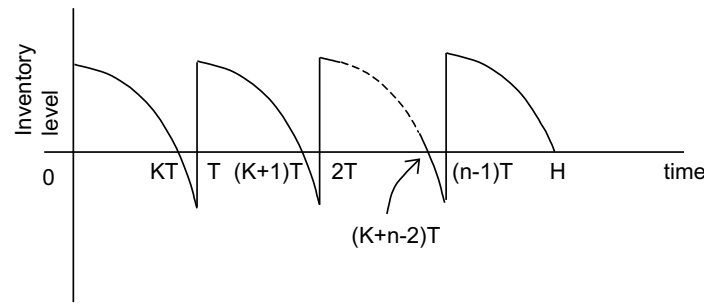
Figure 1:

come under class II. Two separate inflation rates: the internal(company) inflation rate; i_1, and the external (general economy) inflation rate; i_2. These two rates can be estimated by averaging the individual inflation rates of costs in each class.

5. r is the discount rate representing the time value of money.

6. Initially, i.e. at time $t = 0$, c_{11} and c_{12} are internal and external holding costs per unit per time unit; c_{21} and c_{22} are internal and external shortage costs per time unit and p is the external purchase cost.

7. A constant fraction 'θ' ($0 < \theta < 1$) of the on-hand inventory deteriorates per time unit. The deteriorated units can neither be repaired nor be replaced during the period under review.

8. H is the length of the planning horizon. It is divided into n-equal parts of length T, so that $nT = H$. The reorder time points over the time horizon H are jT, j= 0, 1, ..., $(n$-1). Initial and final inventory levels are both zero. We assume that the period for which there is no shortage in each interval $[jT, (j+1)T]$ is a fraction of T and is equal to KT ($0 < K < 1$). Shortages occur at times $(K + j)T$, $j = 0, 1, \ldots\ldots., n$-2 where $jT < (K + j)T < (j + 1)T$. The last replenishment occurs at time $(n$-1)T and shortages are not allowed in the last period $[(n - 1)T, H]$ (see fig. 1)

We want to find the optimal reorder and shortage costs and hence to determine optimal values of discrete variable; n, and continuous variable; K that minimize total cost of an inventory system over the planning horizon $[0, H]$.

3 Mathematical formulation

The present worth of the holding cost over the period $[(j - 1)T, jT]$, j =1,2,..., n-1, is given by

$$H_j = I_{j1} + I_{j2}$$

Where

$$I_{jm} = c_{1m} \int\limits_{(j-1)T}^{(K+j-1)T} \{t - (j - 1)T\}R(t)e^{-R_m t}e^{\theta t}dt$$

$$= ac_{1m} \left[\begin{array}{c} \frac{(K+j-1)Te^{(K+j-1)T(\theta+b-R_m)}}{\theta+b-R_m} - \frac{e^{(K+j-1)T(\theta+b-R_m)}}{(\theta+b-R_m)^2} \\ -\frac{(j-1)Te^{(j-1)T(\theta+b-R_m)}}{\theta+b-R_m} + \frac{e^{(j-1)T(\theta+b-R_m)}}{(\theta+b-R_m)^2} \end{array} \right]$$

$$-ac_{1m}(j-1)T \left[\frac{e^{(K+j-1)T(\theta+b-R_m)}}{\theta+b-R_m} - \frac{e^{(j-1)T(\theta+b-R_m)}}{\theta+b-R_m} \right]$$

where $R_m = r - i_m$, $m = 1, 2$.

The present worth of the holding cost during period $[(n\text{-}1)T, \, nT]$ is

$H_n = I_{n1} + I_{n2}$

Where

$$I_{nm} = c_{1m} \int_{(n-1)T}^{H} \{t - (n-1)T\} \, R(t) e^{-R_m t} e^{\theta t} dt$$

$$= ac_{1m} \left[\frac{He^{(\theta+b-R_m)H}}{\theta+b-R_m} - \frac{e^{(\theta+b-R_m)H}}{(\theta+b-R_m)^2} - \frac{(n-1)Te^{(\theta+b-R_m)H}}{\theta+b-R_m} + \frac{e^{(\theta+b-R_m)(n-1)T}}{(\theta+b-R_m)^2} \right]$$

with $m = 1, 2$.

Hence, the present worth of the total holding cost during the entire time horizon H is

$$C_H = \sum_{j=1}^{n-1} H_j + H_n = \sum_{j=1}^{n-1} \left(\sum_{m=1}^{2} I_{jm} \right) + \sum_{m=1}^{2} I_{nm} \tag{1}$$

Now, the present worth of the shortage cost during $[(K + j-1)T, \, jT]$, $j = 1, 2, \ldots\ldots, n-1$ is

$G_j = J_{j1} + J_{j2}$

Where

$$J_{jm} = c_{2m} \int_{(k+j-1)T}^{jT} (jT - t) R(t) e^{-R_m t} dt$$

$$= ac_{2m} \left[\frac{e^{(b-R_m)jT}}{(b-R_m)^2} - \frac{(1-K)Te^{(b-R_m)(K+j-1)T}}{b-R_m} - \frac{e^{(b-R_m)(K+j-1)T}}{(b-R_m)^2} \right]$$

with $m = 1, 2$.

Hence, the present worth of the total shortage cost during the planning horizon H is given by

$$C_S = \sum_{j=1}^{n-1} G_j = \sum_{j=1}^{n-1} \left(\sum_{m=1}^{2} J_{jm} \right) \tag{2}$$

As there are n – replenishments in the planning horizon H, the present worth of the total ordering cost is

$$C_R = A \sum_{j=0}^{n-1} e^{(i_1-r)jT}$$

$$= A \sum_{j=0}^{n-1} e^{-R_1 jT} \quad \text{where } R_1 = r - i_1$$

$$= \frac{A(1 - e^{-R_1 H})}{1 - e^{-R_1 T}} \tag{3}$$

The present worth of the purchase cost at time $(j-1)T$ for the period $[(j-1)T, (K+j-1)T]$ and at time jT for the period $[(K+j-1)T, jT]$, $j = 1, 2, \ldots\ldots, (n-1)$ is

$$P_{j-1} = pe^{-R_2(j-1)T} \int_{(j-1)T}^{(K+j-1)T} R(t)e^{\theta t} dt + pe^{-R_2 jT} \int_{(K+j-1)T}^{jT} R(t) dt$$

$$= ap\left[\frac{e^{-R_2(j-1)T}}{\theta + b} \left\{ e^{(K+j-1)T(\theta+b)} - e^{(j-1)T(\theta+b)} \right\} + \frac{e^{-R_2 jT}}{b} \left\{ e^{bjT} - e^{b(K+j-1)T} \right\} \right]$$

The present worth of the purchase cost at time $t = (n-1)T$ for the period $[(n-1)T, H]$ is

$$P_{n-1} = pe^{-R_2(n-1)T} \int_{(n-1)T}^{H} R(t)e^{\theta t} dt$$

$$= \frac{ape^{-R_2(n-1)T}}{\theta + b} \left[e^{(\theta+b)H} - e^{(\theta+b)(n-1)T} \right]$$

where $R_2 = r - i_2$.

Hence, the present worth of the total purchase cost is

$$C_P = \sum_{j=1}^{n-1} P_{j-1} + P_{n-1} \tag{4}$$

Using (1) – (4), the present worth of the total cost of the system during the planning horizon is

$$TC(n, K) = C_R + C_P + C_H + C_S \tag{5}$$

4   Solution procedure

The cost function obtained in (5) is a function of the discrete variable n and of the continuous variable K. For a given value of n, the necessary condition for TC to be minimum is

$$\frac{dTC}{dK} = 0$$

where

$$\frac{dTC}{dK} = \left[\begin{array}{l} p \sum_{j=1}^{n-1} \left(e^{-R_2(j-1)T} e^{(K+j-1)T(\theta+b)} - e^{-R_2 jT} e^{b(K+j-1)T} \right) + \\ \\ KT \sum_{j=1}^{n-1} \left(c_{11} e^{(K+j-1)T(\theta+b-R_1)} + c_{12} e^{(K+j-1)T(\theta+b-R_2)} \right) + \\ \\ (K-1)T \sum_{j=1}^{n-1} \left(c_{21} e^{(b-R_1)(K+j-1)T} + c_{22} e^{(b-R_2)(K+j-1)T} \right) \end{array} \right] \tag{6}$$

We can get K, by equating to zero $\frac{dTC}{dK}$ and solving it for K.

i.e. $\frac{dTC}{dK} = 0$

which means to solve

$$XK^2 + YK + Z = 0$$

where

$$X = \sum_{j=1}^{n-1} \sum_{m=1}^{2} \left[\begin{array}{l} c_{1m}aT^3(\theta + b - R_m) + ape^{-R_2(j-1)T}(b+\theta)^2\frac{T^3}{2} - ape^{-R_2jT}\frac{b^2T^3}{2} - \frac{T^4}{2}(b-R_m)^2ac_{2m} \\ +T^3(b-R_m)ac_{2m} \end{array} \right]$$

$$Y = \sum_{j=1}^{n-1} \sum_{m=1}^{2} \left[\begin{array}{l} c_{1m}aT^2 + c_{1m}aT^3(j-1)(\theta+b-R_m) + c_{1m}aT^3(\theta+b-R_m)(j-1) \\ +ape^{-R_2(j-1)T}(b+\theta)T^2 \\ +ape^{-R_2(j-1)T}(b+\theta)^2T^3(j-1) - ape^{-R_2jT}T^2b - ape^{-R_2jT}T^3b^2(j-1) + ac_{2m}T^2 \\ +ac_{2m}T^3(b-R_m)(j-1) - ac_{2m}T^3(b-R_m) - ac_{2m}T^4(b-R_m)^2(j-1) \end{array} \right]$$

$$Z = \sum_{j=1}^{n-1} \sum_{m=1}^{2} \left[\begin{array}{l} 2c_{1m}aT^2(j-1) + c_{1m}aT^3(\theta+b-R_m)(j^2-2j+1) - c_{1m}a(j-1)T^2 + ape^{-R_2(j-1)T}T \\ +ape^{-R_2(j-1)T}(b+\theta)T^2(j-1) + ape^{-R_2(j-1)T}\frac{T^3}{2}(b+\theta)^2(j^2-2j+1) - ape^{-R_2jT}T \\ -ape^{-R_2jT}T^2b(j-1) - ape^{-R_2jT}\frac{T^3}{2}b^2(j^2-2j+1) \\ -ac_{2m}T^2 - ac_{2m}T^3(b-R_m)(j-1) - ac_{2m}\frac{T^4}{2}(b-R_m)^2(j^2-2j+1) \end{array} \right]$$

Hence,

$$K(n) = \frac{-Y \pm \sqrt{Y^2 - 4XZ}}{2X}, \quad n = 1, 2, \ldots\ldots$$

For a given value of n, get K when the values of the other parameters are prescribed. For $n = 1, 2, \ldots\ldots$ we can find the corresponding value of K ($0 < K < 1$) for which TC has to be obtained. The minimum in the above list of TC would be the optimum TC^*. The values of n and K for the minimum TC would be the optimal values for n and K respectively. Since θ is very small θ^2 and its higher powers can be ignored and series approximation in above equation (6) will give value of K for a given n.

5 Numerical example

Consider

$[a, b, p, c_{11}, c_{12}, c_{21}, c_{22}, A, r, i_1, i_2, H, \theta] =$

$[200, 0.03, 5, 0.2, 0.4, 0.8, 0.6, 100, 0.2, 0.08, 0.14, 0.5, 0.04]$

n	TC
53	975313.96
54	995067.39
55	1014809.08
56*	1034564.87*
57	1024328.10
58	1014128.00

It is observed that $TC^* = \$ 1034564.87$ becomes optimum for $n^* = 56$. We have $T^* = H / n^* = 0.008$ yrs.

6 Sensitivity analysis

		n	TC			n	TC
	200	56	1034564.87	H	0.40	69	1034788.49
	205	55	1040072.42		0.50	56	1034564.87
a	210	54	1044617.83		0.55	51	1029122.42
	215	53	1048100.24		0.60	47	1027738.27
	220	52	1050683.99		0.65	43	1010981.62
	225	51	1052203.72		0.70	40	1006147.30
	0.03	56	1034564.87		0.1	54	1030990.88
	0.04	56	1038316.11		0.2	56	1034564.87
b	0.10	55	1040693.11	r	0.3	58	1036918.85
	0.20	54	1058036.99		0.4	60	1037900.04
	0.30	52	1053609.56		0.5	63	1055539.39
	0.40	51	1070249.43		0.6	66	1070983.82
	4	69	1034939.15		0.01	56	1034473.42
	5	56	1034564.87		0.08	56	1034564.87
p	6	47	1027601.45	i_1	0.09	56	1034578.13
	8	36	1023451.39		0.10	56	1034591.41
	9	32	1010578.94		0.15	56	1034658.48
	10	29	1005967.45		0.19	56	1034712.94
	100	56	1034564.87		0.01	58	1026095.79
	110	56	1035108.71		0.07	57	1028790.51
A	120	56	1035652.53	i_2	0.14	56	1034564.87
	130	56	1036196.35		0.15	56	1038255.61
	140	56	1036740.18		0.18	56	1049365.01
	150	56	1037283.10		0.20	56	1056810.64
	0.04	56	1034564.87				
	0.10	55	1032975.48				
θ	0.15	54	1028103.21				
	0.20	53	1022845.49				
	0.30	51	1011596.59				
	0.40	49	999184.10				

It is seen that number of replenishments n is insensitive to changes in parameters A, i_1, i_2; slightly sensitive to changes in a, b, θ and highly sensitive to changes in p, H and r. The optimum total cost is moderately sensitive to changes in a, A, r, i_1, i_2 and highly sensitive to b, p, H and θ.

7 Conclusions

The realistic features of items which are subject to deterioration is incorporated in the study of inventory system. The model deals with exponential time dependent increasing demand. The occurrence of shortages is allowed in the present model. The effects of inflation and the time-value of money are rarely paid attention in most of the literature in the field of inventory

management. Today, the economy of many countries is in the grip of large-scale inflation and consequently sharp decline in the purchasing power of money. In any country, the total investment in inventories in different sectors of the market constitutes a sizable portion of its finance. This being the crucial situation, it is no longer wise to ignore the effects of inflation and the time-value of money in formulating inventory management policies. The model developed here also incorporates this real situation.

The analytical model is developed based on the above stated concepts. The method used of solving the problem is partly analytical and partly computational and is user-friendly. The sensitivity of the solution to changes in the values of different parameters has been discussed.

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