Optimal Pricing and Ordering Policies For deteriorating items under progressive trade credit scheme

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Abstract

In this paper, a mathematical model is developed to formulate optimal pricing and ordering policies when the units in inventory are subject to constant rate of deteriorating and the supplier offers progressive credit periods to settle the account. The concept of progressive credit periods is as follows:

If the retailer settles the outstanding amount by M, the supplier does not charge any interest. If the retailer pays after M but before N (M < N), then the supplier charges the retailer on the un-paid balance at the rate Ic_1. If the retailer settles the account after N, then he will have to pay an interest rate of Ic_2 ($Ic_2 > Ic_1$) on the un-paid balance.

The objective is to maximize the net profit. The decision variables are selling price and ordering quantity. An algorithm is given to find the flow of optimal selling price and ordering policy. A numerical illustration is given to study the effect of offered two credit periods and deterioration on decision variables and the net profit of the retailer.

Resumo

Neste trabalho, um modelo matemático desenvolvido está optimizado para formular políticas de preços e encomendas, quando as unidades do inventário estão sujeitos à taxa constante de deterioração progressiva eo fornecedor oferece crédito períodos de liquidar a conta. O conceito de progressividade de crédito períodos é a seguinte:
Se o varejista apurado o montante pendente por M, o fornecedor não cobra qualquer interesse. Se o revendedor paga após M, mas antes de N (M <N), em seguida, o fornecedor cobra o varejista sobre as un-pago à taxa equilíbrio IC_1. Se o varejista liquidar a conta depois de N, então ele terá que pagar uma taxa de juro de IC_2 ($IC_2 > IC_1$) sobre o saldo un-pagos.
O objetivo é maximizar o lucro líquido. A decisão são variáveis preço de venda e ordenando quantidade. Um algoritmo é determinado a encontrar o fluxo otimizado de preço de venda e ordenação política. A ilustração é dado numérico para estudar o efeito do crédito oferecido dois períodos ea deterioração variáveis e decisão sobre o lucro líquido da varejista.

Keywords: EOQ, Progressive Credit Periods, deterioration, Selling Price, Ordering Policy.

Title: Optimal Preços e Encomenda Políticas Para deterioração progressiva itens sob regime de comércio de crédito

1 Introduction

The Wilson's lot – size model is derived with the assumption that the retailer pays for the goods as soon as it is received by the system. However, in practice, the supplier offers credit period to the retailer to settle his account within the fixed allowable credit period; which encourages retailer to buy more and also attracts more customers. Davis and Gaither (1985) derived a lot – size model when the supplier offers one time opportunity to delay the payments of order, in case, orders for additional units are placed. Shah et al (1988) extended Goyal's (1985) model by allowing shortages. Mandal and Phaujdar (1989) derived a mathematical model by including interest earned from the sales revenue on the stock remaining beyond the settlement period. Shah and Shah (1992) studied inventory model when supplier offers credit period to settle the retailer's account by considering stochastic demand. Jamal et al. (1997) developed an inventory model to allow for shortages under the permissible delay in payments. Shah (1997) derived a probabilistic order-level system with lead-time when delay in payments is permissible. Jamal et al. (2000) formulated a mathematical model when retailer can settle the payment either at the end of the credit period or later incurring interest charges on the un-paid balance for the over-due period. Hwang and Shinn (1997) developed the model for determining the retailer's lot-size and optimal selling price when the supplier permits delay in payments for an order of a product whose demand rate is a function of constant price elasticity.

Arcelus et al. (2001) compared retailer's response to special sales in two strategies viz. price discount and trade credit. Arcelus et al. (2003) derived mathematical model for retailer's maximum profit when supplier offers credit period and/or price discount on the purchase of regular order when units in inventory are subject to constant deterioration. Related articles are by Hadley and Higgins (1973), Kingsman (1983), Chapman et al. (1985), Daellenbach (1986, 1988), Ward and Chapman (1987), Chapman and Ward (1988), Raafat (1991), Wee (1995), Shinn et al. (1996), Chung (1998), Chu et al. (1998), Shah and Shah (2000), Goyal and Giri (2001), Teng (2002) etc.

In this article, an attempt is made to develop mathematical model when units in inventory are subject to constant rate of deterioration and supplier offers two progressive credit periods to the retailer to settle the account. The net profit is maximized with respect to optimal selling and ordering quantity. The effect of deterioration rate of units in inventory system and credit periods on objective function and decision variables are studied using hypothetical numerical example. An algorithm is given to explore the computational flow.

The paper is organized as follows: In section 2, assumptions and notations are given. Section 3 deals with development of mathematical model. In section 4, flowchart is given to search for optimal solution. Analytical results are stated in section 5. The numerical example and observations are given in section 6. The paper concludes with conclusion and bibliography at the end.

2 Assumptions and Notations

The following assumptions are used to develop aforesaid model:

1. The inventory system deals with the single item.
2. The demand is $R(p) = a - bp$, $(a, b > 0, a >> b)$. p denotes selling price of the item during the cycle time and a decision variable.
3. Shortages are not allowed and lead-time is zero.
4. Replenishment is instantaneous.
5. Replenishment rate is finite.
6. If the retailer pays by M, then supplier does not charge to the retailer. If the retailer pays after M and before N $(N > M)$, he can keep the difference in the unit sale price and unit cost in an interest bearing account at the rate of

Ie /unit/year. During [M, N], the supplier charges the retailer an interest rate of Ic_1 /unit/year. If the retailer pays after N, then supplier charges the retailer an interest rate of Ic_2 /unit/year ($Ic_2 > Ic_1$) on un-paid balance.

7. The units on – hand deteriorate at a constant rate θ $(0 \le \theta \le 1)$. The deteriorated units can neither be repaired nor replaced during the cycle time.

The notations are as follows:

- h = The inventory holding cost/unit/year excluding interest charges.
- p = The selling price/unit. (a decision variable).
- C = The unit purchase cost, with C < p.
- A = The ordering cost/order.
- M = The first offered credit period in settling the account without any extra charges.
- N = The second permissible delay period in settling the account N > M.
- Ic_1 = The interest charged per $ in stock per year by the supplier when retailer pays during [M, N].
- Ic_2 = The interest charged per $ in stock per year by the supplier when retailer pays during [N, T].($Ic_2 > Ic_1$)
- Ie = The interest earned/$/year. ($Ic_1 > Ie$)
- T = The replenishment cycle time (a decision variable).
- IHC = Inventory holding cost/cycle.
- PC = Purchase cost / cycle.
- OC = Ordering cost / cycle.
- IE = Interest earned / cycle.
- IC = Interest charged / cycle.
- Q (t) = The on-hand inventory level at time t ($0 \le t \le T$).
- GR = Gross revenue.
- NP(p, T) = Net profit / cycle.
- θ = Deterioration rate

3 Mathematical Formulation

The on-hand inventory depletes due to constant demand R(p) and deterioration of units. The instantaneous state of inventory at any instant of time t is governed by the differential equation

$$\frac{dQ(t)}{dt} + \theta Q(t) = -R(p), \ 0 \le t \le T \tag{3.1}$$

with initial condition Q(0) = Q and boundary condition Q(T) = 0. Consequently, the solution of (3.1) is given by

$$Q(t) = \frac{R(p)(e^{\theta(T-t)}-1)}{\theta}; \ 0 \le t \le T \tag{3.2}$$

and the order quantity is

$$Q = \frac{R(p)(e^{\theta T}-1)}{\theta} \tag{3.3}$$

The cost components per unit time are as follows:

- Ordering cost;

$$\text{OC} = \frac{\text{A}}{T} \tag{3.4}$$

- Inventory holding cost;

$$IHC = \frac{h}{T}\int_0^T Q(t)dt = \frac{h(a-bp)(e^{\theta T}-1-\theta T)}{\theta^2 T} \tag{3.5}$$

- Cost due to deterioration;

$$DC = \frac{C(a-bp)(e^{\theta T}-1-\theta T)}{\theta T} \tag{3.6}$$

- Gross revenue;

$$GR = (p - C)R(p) \tag{3.7}$$

Regarding interest charged and earned, based on the length of the cycle time T, three cases arise:
Case 1: $T \le M$
Case 2: $M < T < N$
Case 3: $T \ge N$
We discuss each case in detail.

Case 1: T ≤ M

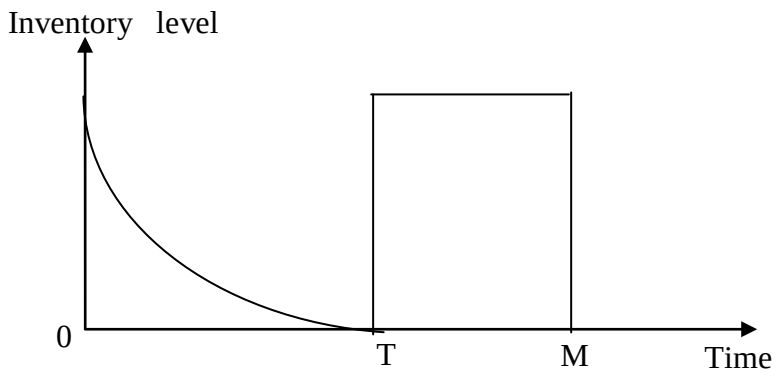


Figure 1: $T \le M$

Here, the retailer sells Q-units during [0, T] and is paying for CQ - units during [0, T] and is paying for CQ in full to the supplier at time $M \ge T$. So interest charges are zero. i.e.

$$IC_1 = 0 \tag{3.8}$$

The retailer sells products during [0, T] and deposits the revenue in an interest bearing account at the rate of I_e/\$/year. In the period, [T, M] the retailer deposits revenue into the account that earns I_e/\$/year. Therefore interest earned per year is

$$IE_1 = \frac{pIe}{T}\left[\int_0^T R(p).t\,d + R(p)T(M-T)\right] = \frac{pIe(a-bp)(2M-T)}{2} \tag{3.9}$$

the net profit; NP_1 is given by

$$NP_1(p, T) = GR - OC - IHC - DC - IC_1 + IE_1 \tag{3.10}$$

p and T are continuous variables. Hence, the optimal values of p and T can be obtained by setting

$$\frac{\partial NP_1(p,T)}{\partial p} = a - 2pb + bC + \frac{(e^{\theta T}-1-\theta T).b}{\theta T}\left(\frac{h}{\theta}+C\right) + \frac{Ie(2M-T)}{2}(a-2pb) = 0 \tag{3.11}$$

and

$$\frac{\partial NP_1(p,T)}{\partial T} = \frac{A}{T^2} - \left(\frac{h}{\theta}+C\right)(a-bp)\left\{\frac{(e^{\theta T}-1)}{T} - \frac{(e^{\theta T}-1-\theta T)}{\theta T^2}\right\} - \frac{pIe(a-bp)}{2} = 0 \tag{3.12}$$

The obtained T = T_1 and $p = p_1$, maximizes the net profit; provided

$$XY - Z^2 < 0 \tag{3.13}$$

where $X = \dfrac{\partial^2 NP_1(p,T)}{\partial p^2} = -2b - Ie\, b(2M - T);$

$$Y = \frac{\partial^2 NP_1(p,T)}{\partial T^2} = \frac{-2A}{T^3} - \left(\frac{h}{\theta} + C\right)(a - bp)\left\{\frac{\theta e^{\theta T}}{T} - \frac{2(e^{\theta T} - 1)}{T^2} + \frac{2(e^{\theta T} - 1 - \theta T)}{\theta T^3}\right\}$$

$$Z = \frac{\partial^2 NP_1(p,T)}{\partial T \partial p} = \frac{(e^{\theta T} - 1)b(h + C)}{T} - \frac{b(e^{\theta T} - 1 - \theta T)}{\theta T^2}\left(\frac{h}{\theta} + C\right) - \frac{Ie(a - 2bp)}{2}$$

Case 2: M < T < N

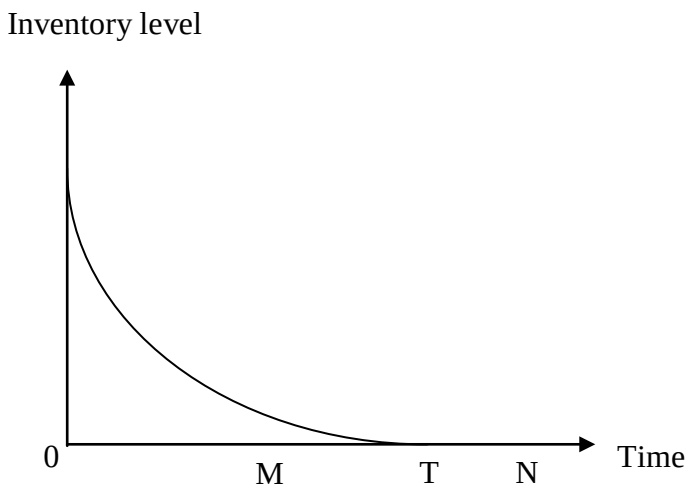


Figure 2: M < T < N

The retailer's sells units and deposits the revenue into an interest bearing account at an interest rate Ie/unit/year during [0, M]. Therefore, interest earned during [0, M] is given by

$$IE_2 = pIe \int_0^M R(p).t\, dt = \frac{1}{2} pIe(a - bp)M^2 \tag{3.14}$$

Buyer has to pay for Q = R(p) T units purchased in the beginning of the cycle at the rate of C \$/unit to the supplier during [0, M]. The retailer sells R(p) M–units at sale price \$ p/unit. So he has generated revenue of p M R(p) plus the interest earned, $IE_{2.1}$, during [0, M]. Two sub-cases may arise:

Sub-case 2.1: Let p R(p) M + $IE_2 \geq$ CQ, i.e. the retailer has enough money to pay for all Q-units procured. Then, interest charges;

$$IC_{2.1} = 0 \tag{3.15}$$

and interest earned; $IE_{2.1}$ per time unit is

$$IE_{2.1} = \frac{IE_2}{T} \tag{3.16}$$

Using equations (3.4) to (3.7), (3.15) and (3.16), the net profit; $NP_{2.1}$ (p, T) is given by

$$NP_{2.1}(p, T) = GR - OC - IHC - DC - IC_{2.1} + IE_{2.1} \tag{3.17}$$

The optimum values of $p = p_{2.1}$ and T = $T_{2.1}$ are solutions of

$$\frac{\partial NP_{2.1}(p,T)}{\partial p} = a-2pb+bC+\frac{(e^{\theta T}-1-\theta T).b}{\theta T}\left(\frac{h}{\theta}+C\right)+\frac{Ie\,M^2(a-2pb)}{2T} = 0 \tag{3.18}$$

and

$$\frac{\partial NP_{2.1}(p,T)}{\partial T} = \frac{A}{T^2} - \left(\frac{h}{\theta}+C\right)(a-bp)\{\frac{(e^{\theta T}-1)}{T} - \frac{(e^{\theta T}-1-\theta T)}{\theta T^2}\} - \frac{pIe(a-bp)M^2}{2T^2} = 0 \tag{3.19}$$

The obtained $p = p_{2.1}$ and T = $T_{2.1}$ maximizes the net profit provided

$$XY-Z^2 < 0 \tag{3.20}$$

Where, $X = \frac{\partial^2 NP_{2.1}(p,T)}{\partial p^2} = -2b - \frac{Ie\,b\,M^2}{T}$;

$$Y = \frac{\partial^2 NP_{2.1}(p,T)}{\partial T^2} = \frac{-2A}{T^3} - \left(\frac{h}{\theta}+C\right)(a-bp)\{\frac{\theta e^{\theta T}}{T} - \frac{2(e^{\theta T}-1)}{T^2} + \frac{2(e^{\theta T}-1-\theta T)}{\theta T^3}\} + \frac{pIe(a-bp)M^2}{2T^3}$$

$$Z = \frac{\partial^2 NP_{2.1}(p,T)}{\partial T \partial p} = \frac{(e^{\theta T}-1)b(h+C)}{T} - \frac{b(e^{\theta T}-1-\theta T)}{\theta T^2}\left(\frac{h}{\theta}+C\right) - \frac{Ie(a-2bp)M^2}{2T^2}$$

Sub case 2.2: Let p R(p) M + IE_2 < CQ

Here, retailer will have to pay interest on un-paid balance U_1 = C R(p) – [p R(p)M + IE_2] at rate of Ic_1 at time M to supplier. The interest to be paid; $IC_{2.2}$ per time unit is:

$$IC_{2.2} = \frac{U_1^2}{pR(p)T} Ic_1 \int_M^T Q(t)dt = \frac{Ic_1 U_1^2 (e^{\theta(T-M)}+M\theta-1-\theta T)}{2\theta^2\, p\, T} \tag{3.21}$$

and interest earned;

$$IE_{2.2} = \frac{IE_2}{T} \tag{3.22}$$

Using equations (3.4) to (3.7), (3.21) and (3.22), the net profit; $NP_{2.2}$ (p, T) is given by

$$NP_{2.2}\ (p, T) = GR - OC - IHC - DC - IC_{2.2} + IE_{2.2} \tag{3.23}$$

The optimum values of $p = p_{2.2}$ and T = $T_{2.2}$ are solutions of

$$\begin{aligned}\frac{\partial NP_{2.2}(p,T)}{\partial p} &= a-2pb+bC+\frac{(e^{\theta T}-1-\theta T).b}{\theta T}\left(\frac{h}{\theta}+C\right)\\ &-\frac{Ic_1U_1(e^{\theta(T-M)}+M\theta-1-\theta T)(-CbT-(a-2bp)M-\frac{Ie\,M^2(a-2bp)}{2})}{\theta^2\, p\, T}\\ &+\frac{Ic_1U_1^2(e^{\theta(T-M)}+M\theta-1-\theta T)}{2\theta^2\, p^2\, T}+\frac{Ie\,M^2(a-2pb)}{2T} = 0\end{aligned} \tag{3.24}$$

and

$$\begin{aligned}\frac{\partial NP_{2.2}(p,T)}{\partial T} &= \frac{A}{T^2}-\left(\frac{h}{\theta}+C\right)(a-bp)\{\frac{(e^{\theta T}-1)}{T}-\frac{(e^{\theta T}-1-\theta T)}{\theta T^2}\} - \frac{Ic_1U_1(e^{\theta(T-M)}+M\theta-1-\theta T)C(a-bp)}{\theta^2\, p\, T}\\ &- \frac{Ic_1U_1^2(e^{\theta(T-M)}-1)}{2\theta\, p\, T} + \frac{Ic_1U_1^2(e^{\theta(T-M)}+M\theta-1-\theta T)}{\theta^2\, p\, T^2} - \frac{pIe(a-bp)M^2}{2T^2} = 0\end{aligned} \tag{3.25}$$

The obtained $p = p_{2.2}$ and T = $T_{2.2}$ maximizes the net profit provided

$$EF-G^2 < 0 \tag{3.26}$$

Where

$$E = \frac{\partial^2 NP_{2.2}(p,T)}{\partial p^2} = -2b - \frac{Ic_1(e^{\theta(T-M)}+M\theta-1-\theta T)(-CbT-(a-2bp)M-\frac{Ie\,M^2(a-2bp)}{2})^2}{\theta^2\,p\,T}$$

$$+\frac{2Ic_1U_1(e^{\theta(T-M)}+M\theta-1-\theta T)(-CbT-(a-2bp)M-\frac{Ie\,M^2(a-2bp)}{2})}{\theta^2\,p^2\,T}$$

$$-\frac{Ic_1U_1(e^{\theta(T-M)}+M\theta-1-\theta T)(2bM+Ie\,b\,M^2)}{\theta^2\,p\,T}-\frac{Ic_1U_1^2(e^{\theta(T-M)}+M\theta-1-\theta T)}{\theta^2\,p^3\,T}-\frac{Ie\,b\,M^2}{T};$$

$$F = \frac{\partial^2 NP_{2.2}(p,T)}{\partial T^2} = \frac{-2A}{T^3}-\left(\frac{h}{\theta}+C\right)(a-bp)\{\frac{\theta e^{\theta T}}{T}-\frac{2(e^{\theta T}-1)}{T^2}+\frac{2(e^{\theta T}-1-\theta T)}{\theta T^3}\}$$

$$-\frac{Ic_1C^2(a-bp)^2(e^{\theta(T-M)}+M\theta-1-\theta T)}{\theta^2\,p\,T}-\frac{2Ic_1U_1(e^{\theta(T-M)}-1)C(a-bp)}{\theta\,p\,T}$$

$$+\frac{2Ic_1U_1(e^{\theta(T-M)}+M\theta-1-\theta T)C(a-bp)}{\theta^2\,p\,T^2}-\frac{Ic_1U_1^2e^{\theta(T-M)}}{2\,p\,T}+\frac{Ic_1U_1^2(e^{\theta(T-M)}-1)}{\theta\,p\,T^2}$$

$$-\frac{Ic_1U_1^2(e^{\theta(T-M)}+M\theta-1-\theta T)}{\theta^2\,p\,T^3}+\frac{pIe(a-bp)M^2}{2T^3}$$

$$G = \frac{\partial^2 NP_{2.2}(p,T)}{\partial T\partial p} = \frac{(e^{\theta T}-1)b(h+C)}{T}-\frac{b(e^{\theta T}-1-\theta T)}{\theta T^2}\left(\frac{h}{\theta}+C\right)$$

$$-\frac{Ic_1(e^{\theta(T-M)}+M\theta-1-\theta T)C(a-bp)(-CbT-(a-2bp)M-\frac{Ie\,M^2(a-2bp)}{2})}{\theta^2\,p\,T}$$

$$+\frac{Ic_1U_1(e^{\theta(T-M)}+M\theta-1-\theta T)C(a-bp)}{\theta^2\,p^2\,T}+\frac{Ic_1U_1(e^{\theta(T-M)}+M\theta-1-\theta T)Cb}{\theta^2\,p\,T}$$

$$-\frac{Ic_1U_1(e^{\theta(T-M)}-1)(-CbT-(a-2bp)M-\frac{Ie\,M^2(a-2bp)}{2})}{\theta\,p\,T}+\frac{Ic_1U_1^2(e^{\theta(T-M)}-1)}{2\theta\,p^2\,T}$$

$$+\frac{Ic_1U_1(e^{\theta(T-M)}+M\theta-1-\theta T)C(a-bp)(-CbT-(a-2bp)M-\frac{Ie\,M^2(a-2bp)}{2})}{\theta^2\,p\,T^2}$$

$$-\frac{Ic_1U_1^2(e^{\theta(T-M)}+M\theta-1-\theta T)}{2\theta^2\,p^2\,T^2}-\frac{Ie(a-2bp)M^2}{2T^2}$$

Case 3: T ≥ N

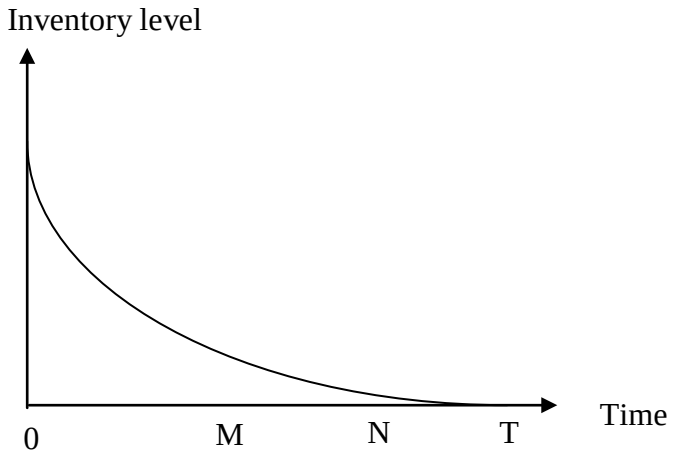


Figure 3: T ≥ N

Based on the total purchase cost, CQ, the total money in account at M is p R(p) M + IE_2 and total money in account at N is p R(p) N + p Ie R(p) $N^2/2$, three sub-cases may arise:

Sub-case 3.1: Let p R(p) M + IE_2 ≥ CQ

This sub-case is same as sub-case 2.1. (Note: Decision variables and objective function are designated by 3.1)

Sub-case 3.2: Let p R(p) M + IE_2 < CQ and

$$p\,R(p)\,(N-M) + \frac{pIeR(p)(N-M)^2}{2} \geq CQ - (p\,R(p)\,M + IE_2)$$

This sub-case coincides with sub-case 2.2. (Note: Decision variables and objective function are designated by 3.2)

Sub-case 3.3: Let p R(p)N + $\frac{pR(p)IeN^2}{2}$ < CQ and

$$p\,R(p)\,(N-M) + \frac{pIeR(p)(N-M)^2}{2} < CQ - (p\,R(p)\,M + IE_2)$$

Here, retailer does not have money in his account to pay off total purchase cost at time N. He will do payment of p R(p) M + IE_2 at M and p R(p) (N–M) + $\frac{pIeR(p)(N-M)^2}{2}$ at N. So, he has to pay interest charges on the un-paid balance U_1 = CQ – (p R(p) M + IE_2) with interest rate Ic_1 during [M, N] and un-paid balance, $U_2 = U_1 - \left(pR(p)\,(N-M) + \frac{pIeR(p)}{2}(N-M)^2 \right)$ with interest rate Ic_2 during [N, T]. Therefore total interest charges; $IC_{3.3}$; per time unit is given by

$$IC_{3.3} = \frac{U_1 Ic_1 (N-M)}{T} + \frac{U_2^2}{pR(p)T} Ic_2 \int_N^T Q(t)dt$$

$$= \frac{U_1 Ic_1 (N-M)}{T} + \frac{Ic_2 U_2^2 (e^{\theta(T-N)} + N\theta - 1 - \theta T)}{\theta^2\, p\, T} \tag{3.27}$$

and interest earned;

$$IE_{3.3} = \frac{IE_2}{T} \tag{3.28}$$

Using equations (3.4) to (3.7), (3.27) and (3.28), the net profit; $NP_{3.3}$ (p, T) is given by

$$NP_{3.3}(p, T) = GR - OC - IHC - DC - IC_{3.3} + IE_{3.3} \tag{3.29}$$

The optimum values of $p = p_{3.3}$ and T = $T_{3.3}$ are solutions of

$$\frac{\partial NP_{3.3}(p,T)}{\partial p} = a - 2pb + bC + \frac{(e^{\theta T} - 1 - \theta T).b}{\theta T}\left(\frac{h}{\theta} + C\right) - \frac{Ic_1(N-M)(-CbT-(a-2bp)M - \frac{Ie\, M^2(a-2bp)}{2})}{T}$$
$$- \frac{2Ic_2U_2(e^{\theta(T-N)} + N\theta - 1 - \theta T)}{\theta^2\, p\, T} + \frac{Ic_2U_2^2(e^{\theta(T-N)} + N\theta - 1 - \theta T)}{\theta^2\, p^2\, T} + \frac{Ie\, M^2(a-2pb)}{2T} = 0 \tag{3.30}$$

and

$$\frac{\partial NP_{3.3}(p,T)}{\partial T} = \frac{A}{T^2} - \left(\frac{h}{\theta} + C\right)(a-bp)\{\frac{(e^{\theta T} - 1)}{T} - \frac{(e^{\theta T} - 1 - \theta T)}{\theta T^2}\} - \frac{Ic_1(a-bp)C(N-M)}{T} + \frac{U_1Ic_1(N-M)}{T^2}$$
$$- \frac{2Ic_2U_2(e^{\theta(T-N)} + N\theta - 1 - \theta T)C(a-bp)}{\theta^2\, p\, T} - \frac{Ic_2U_2^2(e^{\theta(T-N)} - 1)}{\theta\, p\, T} + \frac{Ic_2U_2^2(e^{\theta(T-N)} + N\theta - 1 - \theta T)}{\theta^2\, p\, T^2}$$
$$- \frac{pIe(a-bp)M^2}{2T^2} = 0 \tag{3.31}$$

The obtained $p = p_{3.3}$ and T = $T_{3.3}$ maximizes the net profit provided

$$BK - J^2 < 0 \tag{3.32}$$

where

$$B = \frac{\partial^2 NP_{3.3}(p,T)}{\partial p^2} = -2b - \frac{(2bM + IebM^2)Ic_1(N-M)}{T} - \frac{2Ic_2(e^{\theta(T-N)} + N\theta - 1 - \theta T)\ \%1^2}{\theta^2\, p\, T}$$
$$+ \frac{4Ic_2U_2(e^{\theta(T-N)} + N\theta - 1 - \theta T)\%1}{\theta^2\, p^2\, T} - \frac{2Ic_2U_2(e^{\theta(T-N)} + N\theta - 1 - \theta T)(2bN + Ie\, b\, M^2 + Ie\, b\, (N-M)^2)}{\theta^2\, p\, T}$$
$$- \frac{2Ic_2U_2^2(e^{\theta(T-N)} + N\theta - 1 - \theta T)}{\theta^2\, p^3\, T} - \frac{Ie\, b\, M^2}{T};$$

$$K = \frac{\partial^2 NP_{3.3}(p,T)}{\partial T^2} = \frac{-2A}{T^3} - \left(\frac{h}{\theta} + C\right)(a-bp)\{\frac{\theta e^{\theta T}}{T} - \frac{2(e^{\theta T} - 1)}{T^2} + \frac{2(e^{\theta T} - 1 - \theta T)}{\theta T^3}\}$$
$$+ \frac{2C(a-bp)Ic_1(N-M)}{T^2} - \frac{2U_1Ic_1(N-M)}{T^3}$$
$$- \frac{2Ic_2C^2(a-bp)^2(e^{\theta(T-N)} + N\theta - 1 - \theta T)}{\theta^2\, p\, T} - \frac{4Ic_2U_2(e^{\theta(T-N)} - 1)C(a-bp)}{\theta\, p\, T}$$
$$+ \frac{4Ic_2U_2(e^{\theta(T-N)} + N\theta - 1 - \theta T)C(a-bp)}{\theta^2\, p\, T^2} - \frac{Ic_2U_2^2e^{\theta(T-N)}}{p\, T}$$
$$+ \frac{2Ic_2U_2^2(e^{\theta(T-N)} - 1)}{\theta\, p\, T^2} - \frac{Ic_2U_2^2(e^{\theta(T-N)} + N\theta - 1 - \theta T)}{\theta^2\, p\, T^3} + \frac{pIe(a-bp)M^2}{2T^3}$$

$$J = \frac{\partial^2 NP_{3.3}(p,T)}{\partial T \partial p} = \frac{(e^{\theta T}-1)b(h+C)}{T} - \frac{b(e^{\theta T}-1-\theta T)}{\theta T^2}\left(\frac{h}{\theta}+C\right) + \frac{CbIc_1(N-M)}{T}$$

$$+\frac{(-CbT-(a-2bp)M-\frac{Ie\,M^2(a-2bp)}{2})Ic_1(N-M)}{T^2}$$

$$-\frac{2Ic_2\%2C(a-bp)\%1}{\theta^2\,p\,T}+\frac{2Ic_2U_2\%2C(a-bp)}{\theta^2\,p^2\,T}$$

$$+\frac{2Ic_2U_2\%2Cb}{\theta^2\,p\,T} - \frac{2Ic_2U_2(e^{\theta(T-N)}-1)\%1}{\theta\,p\,T}+\frac{Ic_2U_2^2(e^{\theta(T-N)}-1)}{\theta\,p^2\,T}$$

$$+\frac{Ic_2U_2\%2\%1}{\theta^2\,p\,T^2}-\frac{Ic_2U_2^2\%2}{\theta^2\,p^2\,T^2}-\frac{Ie(a-2bp)M^2}{2T^2}$$

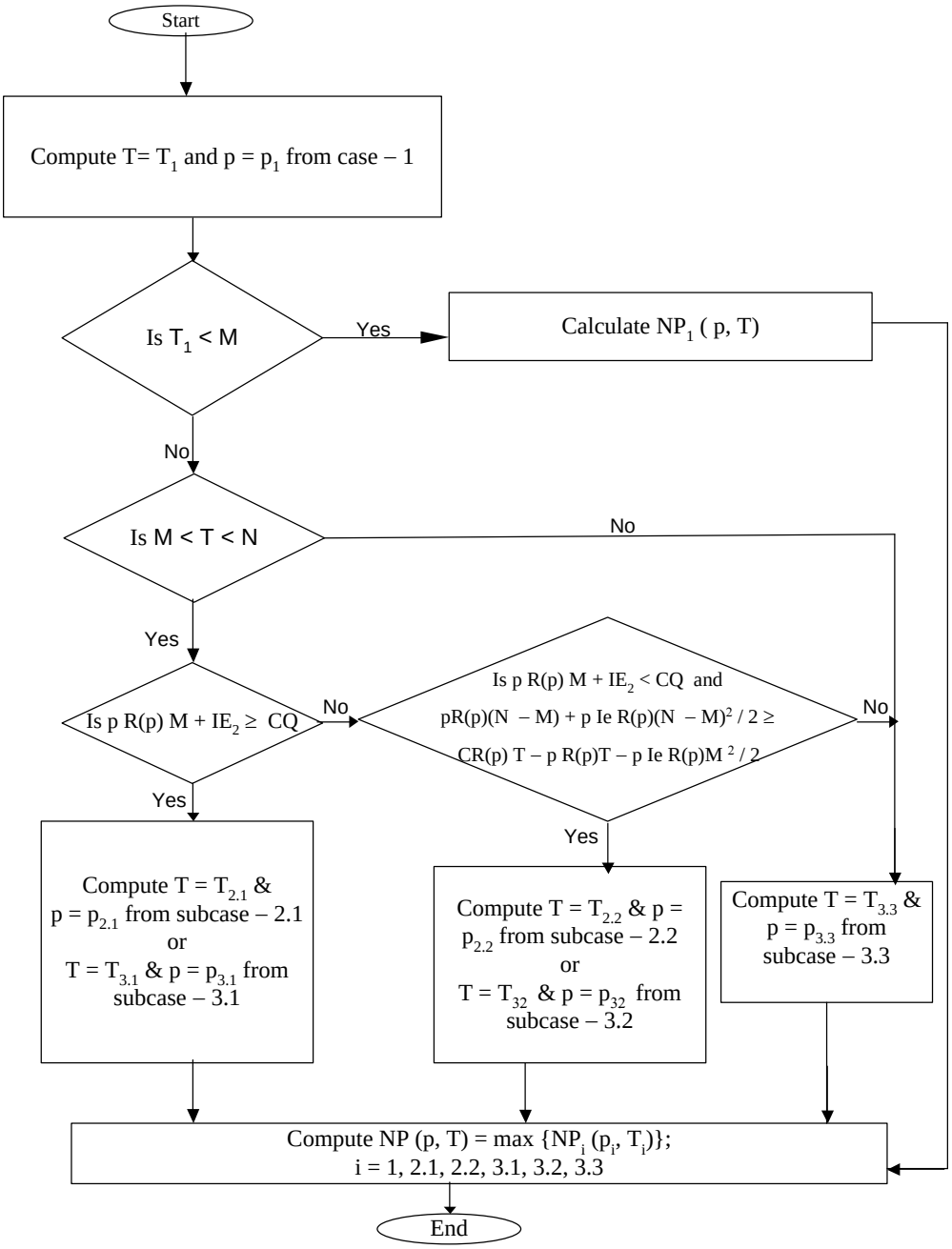
where

$$\%1 = \{-CbT-(a-2bp)M-\frac{Ie\,M^2(a-2bp)}{2}-(a-2bp)(N-M)-\frac{Ie\,(N-M)^2(a-2bp)}{2}\}$$

$$\%2 = (e^{\theta(T-N)}+N\theta-1-\theta T)$$

In the next section, computational flowchart is given to search for optimal solution.

4 Flowchart



5 Theoretical Results

Proposition 5.1: $NP_i(p_i, T_i)$ is maximum for i = 1, 2.1, 2.2, 3.1, 3.2, 3.3.
Proof: It follows from equations (3.13), (3.20), (3.26), (3.32).
Proposition 5.2: For T > N, $NP_{3.3}(p, T)$ is increasing function of M and N.
Proof:

$$\frac{\partial NP_{3.3}(p,T)}{\partial M} = \frac{p(a-bp)(1+M)Ic_1(N-M)}{T} + \frac{U_1 Ic_1}{T} - \frac{2Ic_2U_2\%2}{\theta^2 pT}\left[pIe(a-bp)(N-2M)\right] + \frac{pIe(a-bp)M}{T}$$

$$> 0$$

$$\frac{\partial NP_{3.3}(p,T)}{\partial N} = -\frac{U_1 Ic_1}{T} + \frac{2Ic_2U_2\%2}{\theta^2 pT}\left[p(a-bp)(1+Ie(N-M))\right] + \frac{Ic_2U_2^2(e^{\theta(T-N)}-1)}{\theta pT}$$

$$> 0$$

$$\%2 = (e^{\theta(T-N)} + N\theta - 1 - \theta T)$$

Proposition 5.3: NP_i (p_i, T_i) is a decreasing function of θ.
Proof:

$$\frac{\partial NP_1(p,T)}{\partial\theta} = -\frac{(a-bp)(e^{\theta T}-1)}{\theta}\left(\frac{h}{\theta}+C\right) + \frac{(a-bp)(e^{\theta T}-1-\theta T)}{\theta^2 T}\left(\frac{2h}{\theta}+C\right)$$

$$< 0$$

$$\frac{\partial NP_{2.1}(p,T)}{\partial\theta} = -\frac{(a-bp)(e^{\theta T}-1)}{\theta}\left(\frac{h}{\theta}+C\right) + \frac{(a-bp)(e^{\theta T}-1-\theta T)}{\theta^2 T}\left(\frac{2h}{\theta}+C\right)$$

$$< 0$$

$$\frac{\partial NP_{2.2}(p,T)}{\partial\theta} = -\frac{(a-bp)(e^{\theta T}-1)}{\theta}\left(\frac{h}{\theta}+C\right) + \frac{(a-bp)(e^{\theta T}-1-\theta T)}{\theta^2 T}\left(\frac{2h}{\theta}+C\right) - \frac{Ic_1U_1^2((T-M)e^{\theta(T-M)}+M-T)}{2\theta^2 pT} + \frac{Ic_1U_1^2(e^{\theta(T-M)}+M\theta-1-T)}{\theta^3 pT}$$

$$< 0$$

$$\frac{\partial NP_{3.3}(p,T)}{\partial\theta} = -\frac{(a-bp)(e^{\theta T}-1)}{\theta}\left(\frac{h}{\theta}+C\right) + \frac{(a-bp)(e^{\theta T}-1-\theta T)}{\theta^2 T}\left(\frac{2h}{\theta}+C\right) - \frac{Ic_2U_2^2((T-N)e^{\theta(T-N)}+N-T)}{\theta^2 pT} + \frac{2Ic_2U_2^2(e^{\theta(T-N)}+N\theta-1-T)}{\theta^3 pT}$$

$$< 0$$

6 Numerical Illustration

Using data [A, C, h, a, b] = [200, 20, 0.2, 1000, 10] in appropriate units, we study effect of progressive trade credits and deterioration in following tables:

Table 1: Optimal decision variables when θ = 0.01

N ↓ \ M →		15/365	20/365	25/365
		Ic_1 = 15%	Ic_1 = 16%	Ic_1 = 17%
30/365	Ic_2 = 18%	T = 0.3058 P = 60.59 Q = 120.70 R = 394.09 NP = 15250.85	T = 0.3055 P = 60.57 Q = 120.65 R = 394.34 NP = 15269.16	T = 0.3054 P = 60.54 Q = 120.69 R = 394.56 NP = 15287.32

35/365	Ic_2 = 20%	T = 0.3332 P = 60.56 Q = 131.65 R = 394.44 NP = 15306.35	T = 0.3327 P = 60.53 Q = 131.54 R = 394.69 NP = 15327.64	T = 0.3325 P = 60.51 Q = 131.52 R = 394.93 NP = 15346.31
40/365	Ic_2 = 22%	T = 0.3637 P = 60.53 Q = 143.80 R = 394.71 NP = 15353.73	T = 0.3630 P = 60.50 Q = 143.65 R = 394.96 NP = 15374.53	T = 0.3626 P = 60.48 Q = 143.59 R = 395.22 NP = 15394.32

Table 2: Optimal decision variables when θ = 0.02

N ↓ \ M →		15/365	20/365	25/365
		Ic_1 = 15%	Ic_1 = 16%	Ic_1 = 17%
30/365	Ic_2 = 18%	T = 0.3052 P = 60.60 Q = 120.62 R = 394.04 NP = 15238.73	T = 0.3049 P = 60.57 Q = 120.57 R = 394.28 NP = 15257.04	T = 0.3048 P = 60.55 Q = 120.55 R = 394.50 NP = 15275.19
35/365	Ic_2 = 20%	T = 0.3326 P = 60.56 Q = 131.59 R = 394.38 NP = 15293.16	T = 0.3321 P = 60.54 Q = 131.48 R = 394.63 NP = 15313.44	T = 0.3318 P = 60.51 Q = 131.46 R = 394.87 NP = 15333.11
40/365	Ic_2 = 22%	T = 0.3630 P = 60.54 Q = 143.78 R = 394.64 NP = 15338.30	T = 0.3624 P = 60.51 Q = 143.63 R = 394.90 NP = 15359.17	T = 0.3620 P = 60.48 Q = 143.57 R = 395.16 NP = 15380.91

It is observed that increase in deterioration of units in inventory decreases the net profit and cycle time and increases selling price. The results exhibited in tables coincide with analytical propositions.

7 Conclusion

In this article, an attempt is made to develop an EOQ model in which demand is assumed to be decreasing function of selling price (a decision variable) and units in inventory deteriorate at a constant rate when supplier offers two progressive credit periods, if retailer could not settle his account. An easy-to use computational algorithm is given to search for optimal policy. The observed managerial issues are as follows:

1. Increase in first allowable credit period decreasing the order quantity and increases net profit whereas selling price is insensitive.
2. Increase in extended permissible credit period lowers cycle time and selling price. Net profit decreases significantly.
3. Increase in deterioration rate reduces cycle time and net profit whereas selling price leisurely goes up.

The proposed model can be extended by taking demand as a function of time, product quality, stock etc. It can also be generalized to allow for shortages, partial lost-sales.

8 References

Arcelus, F. J., Shah, Nita H. and Srinivasan, G. (2001) Retailer's response to special sales: Price discount vs. trade credit, OMEGA, Vol 29, No 5, pp. 417 – 428.

Arcelus, F. J., Shah, Nita H. and Srinivasan, G. (2003) Retailer's pricing, credit and inventory policies for deteriorating items in response to temporary price / credit incentives, International Journal of Production Economics,Vol 81-82, pp. 153 – 162.

Chapman, C. B. and Ward, S. C. (1988) Inventory control and trade credit – A Further Reply, Journal of Operational Research Society, Vol 39, pp. 219 – 220.

Chu, P., Chung, K. J., and Lan, S. P. (1998) Economic order quantity of deteriorating items under permissible delay in payments, Computers and Operations Research, Vol 25, pp. 817 – 824.

Chung, K. J.; (1998): A theorem on the determination of economic order quantity under conditions of permissible delay in payments, Computers and Operations Research, Vol 25, pp. 49 – 52.

Daellenbach, H. G.; (1986): Inventory control and trade credit, Journal of Operational Research Society, Vol 37, pp. 525 – 528.

Daellenbach, H. G.; (1988): Inventory control and trade credit – a rejoinder, Journal of Operational Research Society, Vol 39, pp. 218 – 219.

Davis, R. A. and Gaither, N.; (1985): Optimal ordering policies under conditions of extended payment privileges, Management Science, Vol 31, pp. 499 – 509.

Goyal, S. K. and Giri, B. C.; (2001): Recent trends in modeling of deteriorating inventory. European Journal of Operational Research, Vol 134, pp. 1 – 16.

Goyal, S. K.; (1985): Economic order quantity under conditions of permissible delay in payments, Journal of the Operational Research Society, Vol 36, pp. 335 – 338.

Haley, C. W. and Higgins, R. C.; (1973): Inventory policy and trade credit financing, Management Science, Vol 20, pp. 464 – 471.

Hwang, H. and Shinn, S. W.; (1997): Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments, Computers and Operations Research, Vol 24, pp. 539 – 547.

Jamal, A. M., Sarker, B. R. and Wang, S.; (1997): An ordering policy for deteriorating items with allowable shortages and permissible delay in payment, Journal of the Operational Research Society, Vol 48, pp. 826 – 833.

Jamal, A. M. M., Sarker, B. R. and Wang, S.; (2000): Optimal payment time for a retailer under permitted delay of payment by the wholesaler, International Journal of Production Economics, Vol 66, No 1, pp. 59 – 66.

Kingsman, B. G.; (1983): The effect of payment rules on ordering and stockholding in purchasing, Journal of the Operational Research Society, Vol 34, pp. 1085 – 1098.

Mandal, B. N. and Phaujdar, S.; (1989-a): Some EOQ models under permissible delay in payments, International Journal of Management and Systems, Vol 5, No 2, pp. 99 – 108.

Raafat F.; (1991): Survey of literature on continuously deteriorating inventory model, Journal of the Operational Research Society, Vol 42, pp. 27 – 37.

Shah, Nita H. and Shah, Y. K.; (1992): A probabilistic order level system when delay in payments is permissible, presented at Operational Research Society of India, Convention - 25, India.

Shah, Nita H. and Shah, Y.K.; (2000): Literature survey on inventory model for deteriorating items. Economic Annals (Yugoslavia), XLIV, pp. 221 – 237.

Shah, Nita H.; (1997): Probabilistic order level system with lead time when delay in payments is permissible, TOP (Spain), Vol 5, No 2, pp. 297 – 305.

Shah, V. R., Patel, H. C. and Shah, Y. K.; (1988): Economic ordering quantity when delay in payments of order and shortages are permitted, Gujarat Statistical Review, Vol 15, No 2, pp. 51 – 56.

Shinn, S. W., Hwang, H. P. and Sung, S., (1996): Joint price and lot-size determination under conditions of permissible delay in payments and quantity discounts for freight cost, Euporean Journal of Operation Research, Vol 91, pp. 528-542.

Teng, J. T.; (2002): On economic order quantity under conditions of permissible delay in payments, Journal of Operational Research Society, Vol 53, pp. 915 – 918.
Ward, S. C. and Chapman, C. B.; (1987): Inventory control and trade credit – a reply to Daellenbach, Journal of Operational Research Society, Vol 32, pp.1081 – 1084.

Wee, H. M.; (1995): Joint pricing and replenishment policy for deteriorating inventory with declining market, International Journal of Production Economics, Vol 40, pp. 163 – 171.